Organizational Equilibrium with Capital*

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Abstract

This paper proposes a new equilibrium concept – organizational equilibrium – for models with state variables that have a time inconsistency problem. The key elements of this equilibrium concept are: (1) agents are allowed to ignore the history and restart the equilibrium; (2) agents can wait for future agents to start the equilibrium. We apply this equilibrium concept to a quasi-geometric discounting growth model and to a problem of optimal dynamic fiscal policy. We find that the allocation gradually transits from that implied by its Markov perfect equilibrium towards that implied by the solution under commitment, with welfare significantly improved relative to that in the Markov equilibrium. The feature that the time inconsistency problem is resolved slowly over time rationalizes the notion that goodwill is very valuable but has to be built gradually.

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1 Introduction

In this paper we pose an equilibrium concept especially suited for the study of policy settings in macroeconomics where the time inconsistency problem is pervasive and the environment has state variables. Our concept builds upon renegotiation proofness, but adapts it to the challenge of dealing with a dynamic, rather than repeated game. We obtain allocations that are Pareto superior to those of Markov equilibria, but are not supported by trigger-strategy reversion to dominated outcomes.

We argue that equilibria should satisfy three conditions in environments with a sequence of decision makers that see themselves in a similar spot –a form of stationarity even if there are state variables. The first such condition is that any outcome should have the property that no decision maker would rather become an earlier member of the decision making sequence, something that could be achieved by a form of restarting the plans that takes the environment to achieve an allocation. This *no restarting condition* limits the use of trigger strategies as a future punishment. A second condition prevents free riding at the start of the process: no agent can do better by sitting out the system (playing Markov) and waiting for future agents to start a process. This condition is, to our knowledge, new and it prevents jumping to desirable allocations fast. We interpret the implications of this condition as the need for institutions to *slowly earn good will*, like earning a reputation for good behavior without need of unobserved types or triggers. Finally, the third condition imposes optimality within the class of allocations that satisfy the previous two requirements.

Our notion of equilibrium can be seen as a direct extension of that in Prescott and Rios-Rull (2005) and of the notion of *Reconsideration Proofness* in Kocherlakota (1996) to economies with state variables.[1] The extension requires two distinct elements. One is to restrict our attention to a set of environments that display a weak separability property where preferences can be decomposed between a set of actions that we label rescaled actions and the state of the economy. We also

[1] Kocherlakota defines a "state" in his work, but this state only depends on the expectation about current and future actions and is thus purely forward looking. In our case, we define a state as arising from past actions (including possibly past actions of nature, if randomness is present). This is in line with the literature on optimal control and dynamic programming. The role of expectations about current and future actions arises in hybrid environments where some elements of competitive-equilibrium behavior coexist with strategic interactions; we tackle this in Section 5.

provide a strategy to approximate general economies by means of weakly-separable economies to provide a characterization organizational equilibrium via solving for the equilibrium of approximated economies (Kubler (2007)). The other element is the inclusion of an additional equilibrium condition that precludes the delay of the implementation of the equilibrium strategy to future agents. This condition has no bite in an environment without state variables, when the payoff of each player is only affected by her actions and those of *future* players. In contrast, we argue that it is a desirable refinement in the case of economies with state variables. It imposes that the coordination that gives rise to the initial equilibrium is not as generous as to tempt the first players to sit out of it, play Markov, and count on the same coordinating mechanism to arise in the future.

We solve for organizational equilibrium in two benchmark environments. First, we analyze the well studied growth model with quasi-geometric discounting which represents one of the simplest time inconsistency problems (just due to the nature of preferences). Here we show how the economy starts with a very low saving rate and converges to a much higher saving rate, that would have been chosen by any agent if it were to be the constant saving rate for the whole feature. Less simply, but perhaps more interestingly, we solve for the choice of a government that is financed via capital income taxes (other tax instruments can be characterized in essentially similar ways), an environment subject to time inconsistency previously studied by Klein, Krusell, and Ríos-Rull (2008), among others. In both environments the equilibrium allocation is much better (Pareto dominates) that of the Markov perfect equilibrium. The economy slowly moves towards a high/saving or low taxation behavior, that is, the model slowly overcomes the time consistency problem. We interpret this to be a notion of slowly building reputation, without any need for unobservable types. Here what we call reputation is the result of having displayed in the past a form of patience beyond that implied by the behavior in the Markov perfect equilibrium. We think that this type of behavior helps us understand the value that modern institutions such as governments or central banks pose in showing that they have concerns over the long run, and hence do not take actions such as large capital levies or fast inflationary policies that may have been predicted by models where the present is taken to be the initial period.

Our paper is related to various literatures. It studies macroeconomic environments with time-inconsistency features typically characterized in terms of their Markov equilibria (e.g Cohen and Michel (1988), Currie and Levine (1993), Krusell and Ríos-Rull (1996), Klein and Ríos-Rull (2003), Krusell, Kuruscu, and Smith (2010), Klein, Quadrini, and Ríos-Rull (2005), Bassetto and Sargent (2006), Klein, Krusell, and Ríos-Rull (2008), Bassetto (2008)). It also addresses the type of environments previously studied by posing trigger strategies (Chari and Kehoe (1990), Phelan and Stacchetti (2001)). It is clearly related to the class of models that study versions of the quasi-geometric discounting growth model (Strotz (1956), Phelps and Pollak (1968), Laibson (1997), Krusell and Smith (2003), Chatterjee and Eyigungor (2016), Bernheim, Ray, and Yeltekin (2015) among others). Finally, we build on the literature on refinements of subgame perfect equilibrium, particularly in relation to renegotiation proofness (Farrell and Maskin (1989), Kocherlakota (1996), Asheim (1997), Ales and Sleet (2014)). Two other papers are of special relevance. Like us, Nozawa (2014) also tries to extend the notion of a reconsideration-proof equilibrium to economies with state variables. However, his extension imposes too strict requirements and leads to nonexistence of an equilibrium in many applications. By relying on weak separability, our approach allows us to define "state-free" notions of the economic environment and to establish existence. Brendon and Ellison (2018) analyze optimal policy in the Ramsey tradition, but they restrict the planner to choose policies that satisfy a recursive Pareto criterion: this criterion disallows sequences that benefit policymakers in the early periods but are dominated for all policymakers from a given time onward. Like them, we also reject policies that allow early decision makers to dictate future paths that lead to early benefits purely at the expense of future decision makers. Rather than developing an optimality criterion, we propose a solution concept aimed at positive analysis, where implicit cooperation across policymakers at different times builds over time. Because of this different motivation, our "no-restarting condition" is imposed on a period-by-period basis. The presence of state variables causes problems in their environment as well, and our approach based on weak separability could be fruitfully applied there too.[2]

[2]Our approach encompasses the more specific cases introduced by Brendon and Ellison in their latest version to account for state variables.

We start by posing the issues with time inconsistent preferences in the context of the well understood quasi-geometric discounting growth model with log utility and full depreciation in Section 2. We define organizational equilibrium for separable economies in Section 3, where we also describe the connections to game theory. Section 4 describes a class of examples of such economies and we then define a strategy to study non separable economies via this class of approximation using separable economies. We study the implications of organizational equilibrium for public policy in environments with time-consistency problems in Section 5, adapting our concept to hybrid settings of competitive and strategic behavior. Section 6 concludes.

2 Organizational Equilibrium in a Quasi-Geometric Discounting Growth Model

To provide intuition, we explore the concept of organizational equilibrium in a single-agent decision problem with time-inconsistent preferences, the canonical growth model with quasi-geometric discounting, log utility and full depreciation. This allows us to abstract from considerations relating to the competitive equilibrium emerging from the interaction of many agents, which we analyze in Section 5.

Assume that the production function is

$$f(k_t) = k_t^{\alpha},$$

and the period utility function is

$$u(c_t) = \log c_t.$$

The law of motion for the state is

$$k_{t+1} = f(k_t) - c_t.$$

The lifetime utility for the agent at period t is given by

$$\Psi_t = u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(c_{t+\tau}\right).$$

It is easy to see that the agent will disagree with herself in the next period if $\delta \neq 1$. For reasons that will be apparent later, we describe the behavior of the household in terms of its saving rates (note that the feasibility of the choice is independent of the level of capital).

2.1 Traditional Notions of Equilibrium in the Quasi-geometric Discounting Economy

Before we discuss the notion of organizational equilibrium, we first characterize the Ramsey outcome (Section 2.1.1), the differentiable Markov equilibrium (the Markov equilibrium that is the limit of finite economies) (Section 2.1.2), and the best subgame-perfect equilibrium which can be supported by the threat of reversion to a Markov equilibrium (Section 2.1.3).

2.1.1 Ramsey Outcome in the Quasi-geometric Discounting Economy

Suppose that the agent can commit to a particular sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$ at time 0, then the problem of the agent at time 0 is

$$\max_{\{s_t\}_{t=0}^{\infty}} u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t \, u\left(c_t\right),$$

subject to

$$\begin{aligned} k_{t+1} &= s_t k_t^{\alpha}, \\ c_t &= (1 - s_t) k_t^{\alpha}, \\ k_0 & \text{ given.} \end{aligned}$$

This problem can be broken into two components: choosing s_0 and choosing $\{s_t\}_1^{\infty}$. Given s_0, the maximization with respect to future saving rates is a standard recursive problem whose solution has a closed form that is given by the following value function:

$$\Omega(k) = \frac{1}{1-\beta}\left[\log(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\log(\alpha\beta)\right] + \frac{\alpha}{1-\alpha\beta}\log k.$$

This value function is associated with an optimal saving rate which is constant at $s_t = s^R = \alpha\beta$. The Ramsey problem reduces to

$$\max_{s_0} \quad u[(1-s_0)k_0^\alpha] + \delta\,\beta\,\Omega(s_0 k_0^\alpha).$$

The optimal choice of initial saving rate s_0 is

$$s_0 = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}.$$

The initial agent discounts the future more heavily than her future selves, so she is willing to apply a lower saving rate than those in the future, $s_0 < \alpha\beta$. In summary, the sequence of saving rates is

$$s_t = \begin{cases} \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}, & t = 0 \\ \alpha\beta, & t > 0 \end{cases} \tag{2.1}$$

The long run capital stock in the Ramsey problem is $k^R = (\alpha\beta)^{\frac{1}{1-\alpha}}$.[3]

A useful auxiliary problem is the payoff when all selves choose the same constant saving rate. Supposing an agent starting with capital k and the saving rate for herself and all future selves is s, her lifetime utility is given by

$$\frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log s. \tag{2.2}$$

Define the second part related to the saving rate as

$$\mathcal{H}(s) \equiv \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log s. \tag{2.3}$$

The left panel in Figure 1 displays a typical pattern for $\mathcal{H}(s)$. Note that utility is not monotonic in s, nor does it achieve it maximum at the Ramsey long-run solution s^R, since such solution disregards

[3] This level of capital is not strictly a steady state since the Ramsey allocation starting from this level of capital will reduce the level of capital before asymptotically returning to it.

the concern for the short run implied by $\delta < 1$. For comparison, consider the version of $\mathcal{H}(s)$ when

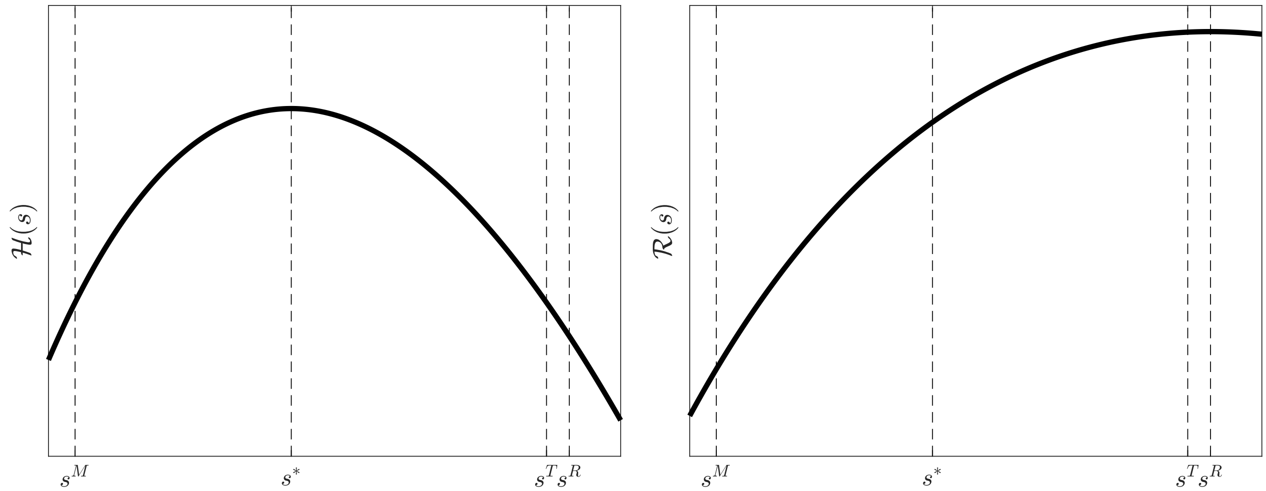


FIGURE 1: Steady State Comparison

$\delta = 1$, where the time-inconsistent problem vanishes. Define $\mathcal{R}(s)$ as

$$\mathcal{R}(s) \equiv \frac{\log(1-s)}{1-\beta} + \frac{\alpha\beta}{(1-\alpha\beta)(1-\beta)} \log s. \tag{2.4}$$

Note that unlike $\mathcal{H}(s)$, $\mathcal{R}(s)$ achieves its maximum at $s^R = \alpha\beta$. Later it will be clear that the functions $\mathcal{H}(s)$ and $\mathcal{R}(s)$ play an important role in understanding the nature of different equilibria.

2.1.2 Markov Equilibrium in the Quasi-geometric Discounting Economy

We focus on the Markov equilibrium which is continuously differentiable, i.e., it satisfies the generalized Euler equation (GEE).[4] Let $g(k)$ denote the policy function for tomorrow's capital k', the GEE is

$$u_c(f(k) - g(k)) = \beta\, u_c\Big(f[g(k)] - g[g(k)]\Big)\Big[\delta f_k[g(k)] + (1-\delta)\, g_k[g(k)]\Big],$$

[4]It remains to be an open question whether the Markov equilibrium is unique or not. As shown in Krusell and Smith (2003), there is a continuum of Markov equilibria in this economy around a steady state. Recent work by Cao and Werning (2018) shows that if production functions are linear the type construction of continuum of equilibria used in Krusell and Smith (2003) would imply a particular restriction of the state space.

which yields

$$g(k) = \frac{\alpha\delta\beta}{1 - \alpha\beta + \delta\alpha\beta} k^{\alpha}.$$

This Markov equilibrium displays a constant saving rate

$$s^M = \frac{\alpha\delta\beta}{1 - \alpha\beta + \delta\alpha\beta}. \tag{2.5}$$

Note that the saving rate in the Markov equilibrium is the same as the first period's saving rate in the Ramsey outcome. The Markov equilibrium has a steady state $k^M = \left(\frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}\right)^{\frac{1}{1-\alpha}} < k^R$. The payoff in the Markov equilibrium for an agent with capital k is given by

$$\Phi^M(k) = \frac{\alpha(1 - \alpha\beta + \delta\alpha\beta)}{1 - \alpha\beta} \log k + \mathcal{H}(s^M). \tag{2.6}$$

2.1.3 Subgame-Perfect Equilibria in the Quasi-geometric Discounting Economy

As discussed in Laibson (1994), by threatening to consume all the resources, any feasible plan can be supported as a subgame-perfect equilibrium. To avoid this unrealistic punishment, we restrict our attention to the set of subgame-perfect equilibria in which the worst payoff is the one delivered by the Markov equilibrium characterized above, $\Phi^M(k)$. The best such outcome is

$$\max_{\{s_t\}_{t=0}^{\infty}} \quad u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t u(c_t) \qquad \text{s.t.} \tag{2.7}$$

$$k_{t+1} = s_t k_t^{\alpha}, \tag{2.8}$$

$$c_t = (1 - s_t) k_t^{\alpha} \tag{2.9}$$

$$u(c_t) + \delta \sum_{j=1}^{\infty} \beta^t u(c_{t+j}) \geq \Phi^M(k_t) \tag{2.10}$$

$$k_0 \text{ given }. \tag{2.11}$$

We relegate the detailed analysis of this equilibrium to Appendix A where we characterize its solution and establish that it has a stationary form. Essentially, the results can be partitioned into two scenarios. In the first case time inconsistency is relatively weak, *i.e.* δ is large enough and the folk theorem applies. As a result, the Ramsey outcome can be supported in a subgame-perfect equilibrium. In the second case, δ is relatively low, and the temptation to deviate is strong, in which case the constraint (2.10) binds forever. To induce future selves not to revert to the Markov equilibrium, the saving rate has to be lower than that in the Ramsey problem. Combining the two cases, the outcome can be summarized as

$$s_t = \begin{cases} s^R, & \mathcal{H}\left(s^R\right) \geq \mathcal{H}\left(s^M\right) \\ s^T, & \mathcal{H}\left(s^R\right) < \mathcal{H}\left(s^M\right) \end{cases} \tag{2.12}$$

for $t > 0$ and $s_0 = s^M$, where s^T solves

$$s^T = \max_s \left\{s : \mathcal{H}(s) = \mathcal{H}\left(s^M\right)\right\} \tag{2.13}$$

Note that when the constraint (2.10) is binding in the steady state, the saving rate is not the same as the Markov equilibrium. Each agent at time $t > 0$ attains the same payoff in this equilibrium as if the Markov equilibrium were played *from then on*, but the initial player achieves strictly higher utility, and all players that move at $t > 1$ benefit from the higher saving rate that prevailed *in the past.*[5]

2.2 Organizational Equilibrium in the Quasi-geometric Discounting Economy

Before we start describing the notion of organizational equilibrium, we establish an important property of this economy, that preferences of agents over the inherited capital stock and any sequence of savings rates display separability, *i.e.* that the utility can be written as a function of the initial capital and of an aggregate that depends only on the sequence of savings rates (Section 2.2.1). Then we define an organizational equilibrium in terms of savings rates, and we construct it, therefore establishing existence in Section 2.2.2.

[5] The maximization problem only takes into account the utility of the time-0 agent; the benefit accruing to later players is a byproduct of this agent's preferred outcome, which involves high saving rates from period 1 onwards.

2.2.1 Separability

Consider a sequence of savings rates $\{s_t\}_{t=0}^{\infty}$ and an initial capital stock k_0. The implied sequence of capital stocks, $\{k_t\}_{t=0}^{\infty}$, has the property that $k_t = k_0^{\alpha^t}\Pi_{j=0}^{t-1}s_j^{\alpha^{t-j-1}}$. Accordingly, the lifetime utility for the agent with capital k_0 is

$$
\begin{aligned}
& U(k_0, s_0, s_1, \ldots) \\
&= \log(1-s_0)k^{\alpha} + \delta\sum_{j=1}^{\infty}\beta^j\log[(1-s_j)k_j^{\alpha}] \\
&= \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_0 + \log(1-s_0) + \delta\sum_{j=1}^{\infty}\beta^j\log[(1-s_j)\Pi_{\tau=0}^{j-1}s_{\tau}^{\alpha^{j-\tau}}] \\
&= \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_0 + \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_0) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_j) + \frac{\alpha\beta}{1-\alpha\beta}\log(s_j)\right).
\end{aligned}
$$

The same logic follows for the date t agent which means that its lifetime utility, or total payoff, is the sum of a term that depends on the period t capital and a term that depends only on saving rates of periods t and after. We write it compactly as

$$
\underbrace{U(k_t, s_t, s_{t+1}, \ldots)}_{\text{total payoff}} = \underbrace{\phi\log k_t}_{\text{capital payoff}} + \underbrace{V(s_t, s_{t+1}, \ldots)}_{\text{action payoff}} \tag{2.14}
$$

We also write more compactly the total payoff into the term that depends on the state and the term that depends on the subsequent actions

$$
v(k, V) := \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_0 + V
$$

where

$$
V(s_0, s_1, \ldots) := \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_0) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_j) + \frac{\alpha\beta}{1-\alpha\beta}\log(s_j)\right).
$$

In this economy, the preferences as of time t are separable in the level of capital k_t and the sequence of current and future saving rates. In fact, that the terms in k_0 and V are additive implies a *strong* form of separability. For our purposes, only a weak form of separability is required as we will see in Section 3.

2.2.2 Discussion of Organizational Equilibrium

We now look at the features that the organizational equilibrium should have. We exploit separability to specify how to run comparisons across agents with differing initial conditions: specifically, we impose that agents factor out the component of utility arising from initial capital, and evaluate proposals based on the sequence of saving rates alone, looking at the subutility V. On this subutility, we impose the requirements that no agent would prefer being a previous member of the sequence and that no agent would have an incentive to wait for a proposal to be implemented starting from the next period. Formally,

Definition 1. *A sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$ is organizationally admissible if*

1. *$V(s_t, s_{t+1}, s_{t+2}, \ldots)$ is (weakly) increasing in t.*
2. *The first agent has no incentive to delay the proposal.*

$$V(s_0, s_1, s_2, \ldots) \geq \max_s V(s, s_0, s_1, s_2, \ldots)$$

Within organizationally admissible sequences, any sequence that attains the maximum of $V(s_0, s_1, s_2, \ldots)$ is an organizational equilibrium*.*

We now start characterizing the organizational equilibrium.

Proposition 1. *There exist organizational equilibria. In any such equilibrium, the evolution of the* *saving rate is given recursively by the proposal function* q^*

$$s_t = q^*(s_{t-1}) = 1 - \exp\left\{ \frac{-(1-\beta)V^* + \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_{t-1} + \log(1-s_{t-1})}{\beta(1-\delta)} \right\} \tag{2.15}$$

where the fixed point saving rate $s^* = q^*(s^*)$ *and the maximum utility* V^* *are given by*

$$s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta} \qquad \text{and} \tag{2.16}$$

$$V^* = \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s^*) + \frac{\alpha\delta\beta}{(1-\beta)(1-\alpha\beta)}\log s^*. \tag{2.17}$$

Equilibria differ by their initial saving rate, which belongs to the interval

$$s_0 \in \left[\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}, q^*\left(\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta} \right) \right]. \tag{2.18}$$

Proof. Suppose the initial agent with capital $k_0 = k$ proposes a sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$, which yields a sequence of capital $\{k_\tau\}_{\tau=0}^{\infty}$. Consider a subsequence of the proposed saving rates from time t on, $\{s_\tau\}_{\tau=t}^{\infty}$. This is the sequence of saving rates that will be used by the agent with capital k_t. The lifetime utility for the agent with capital k_t is

$$U_t(k_t, \{s_\tau\}_{\tau=t}^{\infty}) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + V_t,$$

where

$$V_t = \log(1-s_t) + \frac{\delta\alpha\beta}{1-\alpha\beta}\log(s_t) + \delta\sum_{j=1}^{\infty}\beta^k\left(\log(1-s_{t+j}) + \frac{\alpha\beta}{1-\alpha\beta}\log(s_{t+j})\right). \tag{2.19}$$

The link between V_t and V_{t+1} is given by

$$V_t - \beta V_{t+1} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - \beta(1-\delta)\log(1-s_{t+1}). \tag{2.20}$$

We will proceed by guessing and verifying. We first ignore condition 2 in the definition of organizationally admissible sequence, which will imply that a proposal has to be that $V_t = \overline{V}$. This is because

V_t has to be weakly increasing, and the proposer can be better off by copying future agents' saving rate sequence if $V_t > V_0$ for some t. Later on, we will verify that condition 2 can be satisfied by a particular selection of a saving rate sequence.

The constant action payoff restriction, $V_t = \overline{V}$, simplifies equation (2.20) to

$$(1-\beta)\overline{V} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - \beta(1-\delta)\log(1-s_{t+1}). \tag{2.21}$$

Equation (2.21) has two implications. First, it establishes a recursive relationship for saving rates between two consecutive agents. Second, it provides us a way to select the saving rates sequence that yields the highest utility.

Denote by $s = q(s^-;\overline{V})$ as the proposal function that specifies the current generation's saving rate as a function of last generation's saving rate:

$$q(s^-;\overline{V}) = 1 - \exp\left\{\frac{-(1-\beta)\overline{V} + \frac{\delta\alpha\beta}{1-\alpha\beta}\log s^- + \log(1-s^-)}{\beta(1-\delta)}\right\}. \tag{2.22}$$

If the initial agent wants to make a proposal, it will choose the highest $\overline{V}$ possible. Given a particular $\overline{V}$, the fixed point of the function $q(s^-;\overline{V})$ solves

$$(1-\beta)\overline{V} = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) - \beta(1-\delta)\log(1-s). \tag{2.23}$$

Also note that

$$\frac{\delta\alpha\beta}{1-\alpha\beta}\log s + \log(1-s) - \beta(1-\delta)\log(1-s) \in (-\infty, V^*],$$

where V^* is given by

$$s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta}, \tag{2.24}$$

$$V^* = \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s^*) + \frac{\alpha\delta\beta}{(1-\beta)(1-\alpha\beta)}\log s^*. \tag{2.25}$$

If the initial agent chooses $\overline{V} > V^*$, then there is no fixed point for $q(s^-; \overline{V})$. Meanwhile, $q(s^-; \overline{V}) > s^-$ when $\overline{V} > V^*$, and s will converge to 1 at a rate which will make the sum in equation (2.19) diverge to $-\infty$, which cannot be optimal. Therefore, the optimal choice is V^*. The optimal proposal function associated with V^* is $q^*(s^-) = q(s^-; V^*)$.

If we ignore condition 2, there are many valid proposals that the initial agent could make; as an example, a constant saving rate s^* would be one of them. However, this constant saving rate sequence $\{s^*\}$ will violate condition 2. If the initial agent waits for the next generation to propose this constant saving rate sequence, then the initial agent can choose the Markov saving rate $\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}$, which yields a higher utility. This will be the case for all proposals in which

$$s_0 \in \left(q^* \left(\frac{\alpha\delta\beta}{1 - \alpha\beta + \alpha\delta\beta} \right), s^* \right].$$

Furthermore, simple but tedious algebra shows that $ds/ds^- = 1$ and $d^2s/d(s^-)^2 > 0$ at $s = s^*$ when $\overline{V} = V^*$. s^* is thus a semi-stable steady state: it is stable from the left, but unstable from the right. Proposals in which $s_0 > s^*$ would lead the difference equation to converge to 1, which is ruled out by the same argument which we used to establish that values above V^* are unattainable.

Equation (2.22) attains a minimum when s^- is at the Markov saving rate and is strictly decreasing below this value, converging to 1 as s^- converges to 0. Hence, there exists a value $\underline{s}$ such that $q(s) > s^*$ for $s < \underline{s}$: this is a lower value on the initial saving rate, since once again the sequence would otherwise yield arbitrarily negative utility.

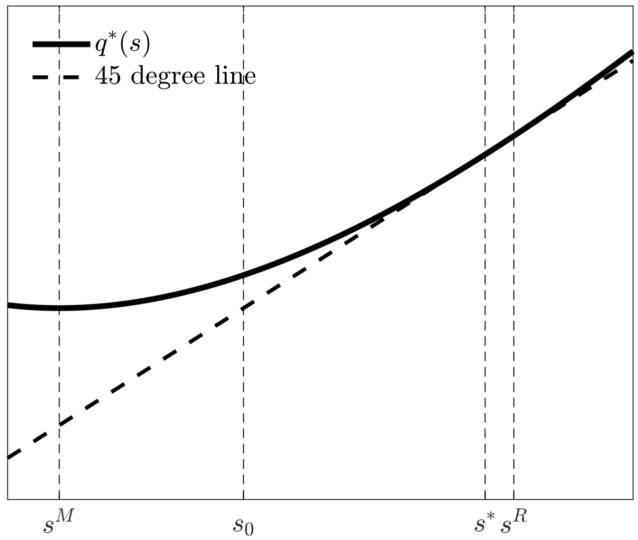
The set of organizational equilibria is given by the set of sequences which satisfy (2.16), (2.17), and (2.22), and which have

$$s_0 \in \left[\underline{s}, q^* \left(\frac{\alpha\delta\beta}{1 - \alpha\beta + \alpha\delta\beta} \right) \right].$$

□

While there are many organizational equilibria, all of whom give the same utility to the first gener-

FIGURE 2: Proposal Function $q^*(s)$



ation, the equilibrium in which

$$s_0 = q^* \left(\frac{\alpha\delta\beta}{1 - \alpha\beta + \alpha\delta\beta} \right)$$

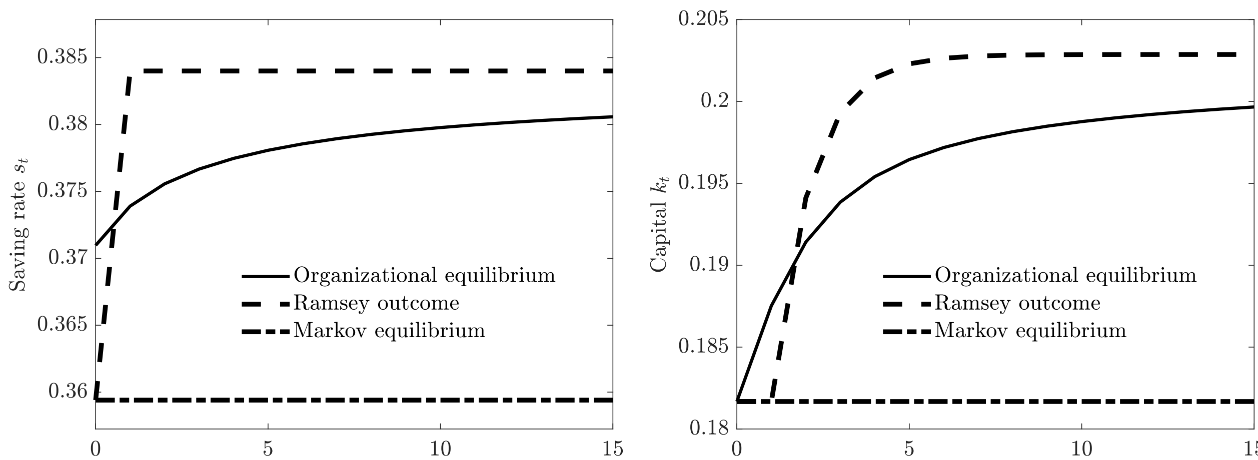
yields the highest utility for all subsequent generations, and is therefore the most appealing.

In any organizational equilibrium, time inconsistency is gradually overcome through time: at least from period 2 on, the saving rate exceeds that of the Markov equilibrium, and a virtuous cycle is started, with a monotonic increase which converges to s^*. Initial saving is limited by the temptation to let the next generation start the virtuous cycle. This temptation diminishes in subsequent periods, since restarting the virtuous cycle from scratch implies giving up on the accumulated effect of previous increases in s_t. Note that s^* is below the long-term savings rate of the Ramsey outcome, no matter how close to 1 δ is (as long as it is strictly less than 1): while the equilibrium path converges to the Ramsey outcome as $\delta \to 1$, it never coincides with it, and the folk theorem does not apply. The saving rate s^* is the preferred constant saving rate of the agents, weighing their current patience with their future impatience. This is illustrated in Figure 2.

2.3 Comparison with Other Equilibria

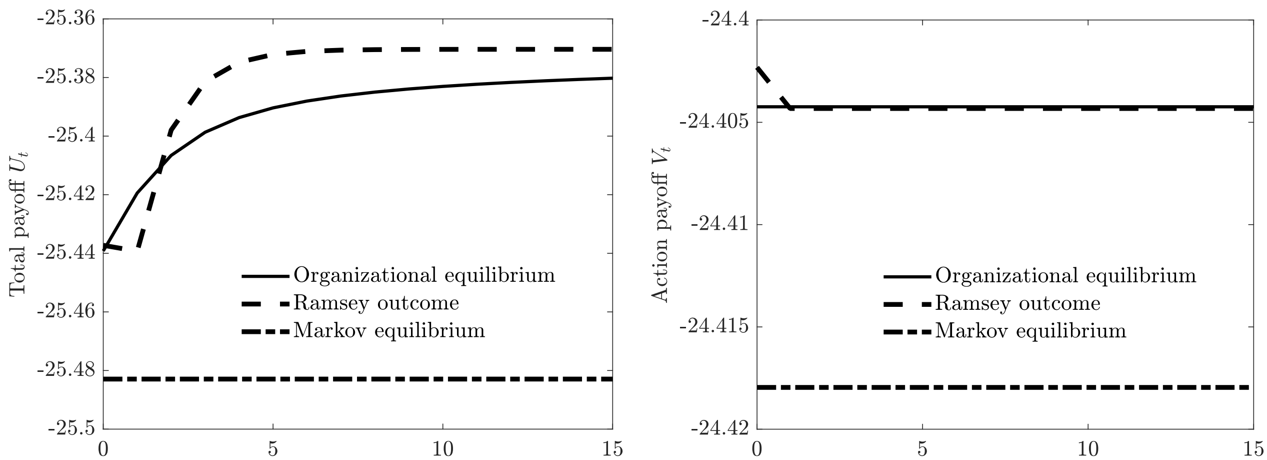
We now compare the properties of the organizational equilibrium of the sequence of capital and the lifetime utilities with those in the Ramsey outcome, the Markov equilibrium and the best (from the point of view of the preferences of the time zero agent) subgame-perfect equilibrium supported by the threat of reversion to Markov.

FIGURE 3: Transition Path I: Allocation



We first turn the transition paths of different equilibria. We assume that the initial capital stock is the steady state capital stock in the Markov equilibrium, i.e., $k_0 = k^M$. The value of δ we have specified in this example is large enough that the Ramsey outcome can be supported by the threat of reversion to Markov. Figure 3 displays the transition paths for the saving rate s_t and capital k_t. In the Markov equilibrium, the capital stock remains unchanged at its steady-state level which we assumed as a starting point. The Ramsey outcome features the same saving rate as the Markov equilibrium in the first period, so that the capital stock remains the same at the beginning of the second period. From the second period onwards the saving rate increases to s^R permanently. The sequence of saving rates in the organizational equilibrium is induced by the proposal function $s_{t+1} = q^*(s_t)$. Particularly, the saving rate in the first period is $s_0 = q^*\left(s^M\right) > s^M$, and the capital is initially higher than in the Ramsey allocation. Over time, the saving rates increases gradually and converge to $s^* < s^R$. Asymptotically, capital in the organizational equilibrium settles between the Ramsey outcome and the Markov equilibrium.

FIGURE 4: Transition Path II: Payoff



Now we turn to the welfare comparison. Given a particular sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$, based on the analysis in the last section, the lifetime utility for generation t can be written as

$$\underbrace{U_t(k_t, \{s_\tau\}_{\tau=t}^{\infty})}_{\text{total payoff}} = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + \underbrace{V_t}_{\text{action payoff}} .$$

The total payoff U_t and the action payoff V_t are depicted in Figure 4. The total payoff in the Markov equilibrium is the lowest during the entire transition, which is the result of both the lowest capital stock and action payoff.

The comparison between the Ramsey outcome and the organizational equilibrium is more subtle. In the first period, the total payoff in the Ramsey outcome is higher than that in the organizational equilibrium: this has to happen by definition, since the Ramsey outcome maximizes the total payoff from the perspective of period 0. In the following period, the comparison reverses, and the total payoff in the organizational equilibrium is actually higher than the Ramsey outcome. This happens both because the initial generation accumulates additional capital, and because the organizational equilibrium does not impose as high a saving rate, allowing for some indulgence for the short-run impatience that arises in the second period. Our notion of organizational equilibrium treats initial capital as a bygone, factoring it out of the payoff that is relevant in computing the equilibrium itself; however, it captures the notion that the initial agent is not privileged compared to future decision makers and cannot impose on them sacrifices that she has not undertaken. For this reason, when we focus on V_t, an organizational equilibrium redistributes from the initial agent to all future decision makers. When comparing the total payoff, after period 0, early decision makers benefit both from a higher capital level and a higher action payoff, while eventually capital falls below the Ramsey outcome and late generations lose from this.

In terms of the steady state capital level, Figure 5 shows how it changes with δ. The capital stock in the organizational equilibrium (k^O), the Markov equilibrium (k^M), and the Ramsey outcome (k^R) are related by the following simple ratios:

$$\frac{k^O}{k^R} = \left(\frac{1}{\frac{1}{\delta} - \beta(1-\alpha\beta+\alpha)\left(\frac{1}{\delta}-1\right)} \right)^{\frac{1}{1-\alpha}} \text{ and } \frac{k^M}{k^O} = \left(1 - \frac{\beta(1-\delta)(1-\alpha\beta)}{1-\alpha\beta+\delta\alpha\beta}\right)^{\frac{1}{1-\alpha}}.$$

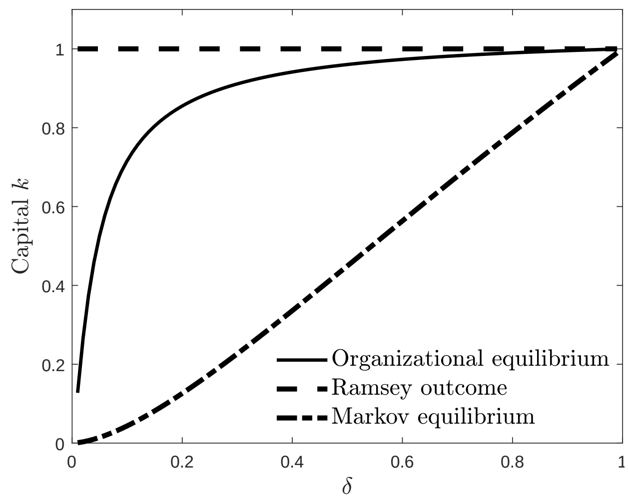
In the Ramsey outcome, the stationary allocation is independent of δ, and we normalize it to 1. As can be seen from the Figure, as δ decreases, the capital stock in the Markov equilibrium decreases faster than in the organizational equilibrium.

3 Organizational Equilibrium: A General Definition

We proceed now to define organizational equilibrium in a more general manner. The spirit of organizational equilibrium is that the solution concept should not treat the current decision maker more favorably than future ones. This notion requires some form of stationarity.

Consider a generic environment of sequential decision makers (typically those that have a time-consistency problem) where there is a physical state variable $k \in K$. Specifically, given the current

Figure 5: Comparison of Stationary Allocations



level of k, the agent making a decision will choose an action a from a set A. The state evolves according to $k_{t+1} = F(k_t, a_t)$. Preferences for an agent making decisions in period t are given by $U(k_t, a_t, a_{t+1}, a_{t+2}, \ldots)$. The first assumption is that functions U and F are independent of calendar time, which allows meaningful welfare comparisons across decision makers.

In the absence of a state variable, the economy looks the same starting at any time t, and "not treating more favorably" the current decision maker readily translates to not offering to her higher utility than to other decision makers. However, when a state variable is present, imposing the same utility becomes an unnatural restriction: as an example, as capital evolves, the sequences of consumption which can be supported by the given capital change. We follow here an alternative approach by restricting the environments that we study to those in which the utility is weakly separable between the state and the sequence of actions, such that the preference ordering over sequences actions is independent of the initial state. It is then natural to require that the equilibrium choice of actions be independent of the state. This amounts to a form of weak separability in terms of utility between the state and the actions. Formally the assumptions on the environment that we make are:

Assumption 1. *1. At any point in time t, the set of feasible actions A is independent of the state*

k_t;

2. *U is weakly separable in k and in $\{a_s\}_{s=0}^{\infty}$, i.e., there exist functions $v : K \times \mathbb{R} \to \mathbb{R}$ and $V : A^{\infty} \to \mathbb{R}$ such that*

$$U(k, a_0, a_1, a_2, \ldots) \equiv v(k, V(a_0, a_1, a_2, \ldots)). \tag{3.1}$$

and such that v is strictly increasing in its second argument.

Assumption 2. *V is in turn weakly separable in a_0 and $\{a_s\}_{s=1}^{\infty}$, i.e., there exist functions $\widetilde{V} : A \times \mathbb{R} \to \mathbb{R}$ and $\widehat{V} : A^{\infty} \to \mathbb{R}$ such that, for all sequences $(a_0, a_1, a_2, \ldots) \in A^{\infty}$,*

$$V(a_0, a_1, a_2, ...) = \widetilde{V}(a_0, \widehat{V}(a_1, a_2, \ldots)),$$

with $\widetilde{V}$ strictly increasing in its second argument.

Sometimes the original problem does not satisfy Assumption 1, but it is possible to rescale actions in such a way that it does. As an example, the original specification of the saving problem with quasi-geometric discounting does not satisfy Assumption 1 if we define the action to be consumption: the feasible set of consumption levels depends on initial capital.[6] Formally, suppose that the set of feasible actions at any capital level k is $\tilde{A}(k) \subseteq \tilde{A}$ and that preferences are given by $\tilde{U}(k_t, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots)$. Our construction still applies as long as it is possible to find a set of actions A and a function γ such that $\tilde{a} = \gamma(a, k)$ and that Assumption 1 holds for A, where

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) \equiv \tilde{U}(k, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots),$$

and where for $t \geq 0$, $\tilde{a}_t$ is computed recursively as

$$\begin{aligned} \tilde{a}_t &= \gamma(a_t, k_t), \\ k_{t+1} &= F(k_t, \tilde{a}_t). \end{aligned} \tag{3.2}$$

[6] Note that weak separability automatically fails if certain actions are only feasible for some levels of capital, since, holding actions fixed, the left-hand side of (3.1) would then be well defined for some values of k and not for others.

We are now ready to define organizational equilibrium.

Definition 2. *A sequence of actions $\{a_t\}_{t=0}^{\infty}$ is organizationally admissible if it satisfies the following requirements:*

1. *$V(a_t, a_{t+1}, a_{t+2}, \ldots)$ is (weakly) increasing in t; this condition ensures that subsequent agents would not choose to rewind time.*
2. *The first agent has no incentive to delay the proposal.*

$$V(a_0, a_1, a_2, \ldots) \geq \max_{a \in A} V(a, a_0, a_1, a_2, \ldots); \tag{3.3}$$

Within organizationally admissible sequences, any sequence that attains the maximum of $V(a_0, a_1, a_2, \ldots)$ is an organizational equilibrium*.*

3.1 Game-Theoretic Foundations and Relation to Other Equilibrium Notions

In this section, we discuss the connection between an organizational equilibrium and related notions of equilibria in games. Our notion is most closely related to Kocherlakota's (1996) reconsideration-proof equilibrium. While the two notions are very similar, two main differences emerge, which we will discuss in turn:

- By exploiting weak separability, an organizational equilibrium extends the notion of reconsideration proofness to dynamic games, rather than purely repeated games;
- In an environment without state variables, all reconsideration-proof equilibria have the same value for all players. This is no longer the case for organizational equilibria. Our no-delaying condition takes into account the role of the state variable to select a subset of the reconsideration-proof equilibria. This selection is in line with the original motivation of renegotiation (and reconsideration) proofness, but it imposes it in a limited way which still allows for existence of an equilibrium.

The general setup that we introduced in this section represents a game, with an infinity of players indexed by the time at which they act (0,1,...), each of whom has preferences given by (3.1). At each time t, the history of play is given by $h^t := (a_0, a_1, ...a_{t-1})$, with $h^0 := \emptyset$. A strategy σ_t for player t is a mapping from the set of time-t histories, H^t, to the set of actions A. A strategy profile is a sequence of strategies, one for each player: $\sigma := (\sigma_0, \sigma_1, ...)$. As usual, it is also convenient to define a continuation strategy after history h^t, $\sigma|_{h^t}$, represented by the restriction of $(\sigma_t, \sigma_{t+1}, ...)$ to the histories following h^t. So far, we have defined an organizational equilibrium only by its equilibrium path. The simplest way to fully specify the game strategy that supports it as a subgame-perfect equilibrium is to use the no-delaying condition (3.3) and set $\sigma|_{h^t} = \sigma$ whenever

$$a_{t-1} \neq \sigma_{t-1}(h^{t-1}), \tag{3.4}$$

where $h^t = (h^{t-1}, a_{t-1})$. According to this strategy, the equilibrium of the game prescribes restarting from the equilibrium path of period 0 whenever a deviation occurs. This is not the only possibility; as an example, the function q^* defined in (2.15) can be used in the quasi-geometric saving problem to recursively generate an alternative strategy (not just an equilibrium path) that supports the organizational equilibrium as a subgame-perfect equilibrium.

When there are no state variables the game presented here is encompassed by those considered in Kocherlakota (1996).[7] Kocherlakota analyzes a purely forward-looking environment, in which the payoff accruing to the player in period-t only depends on the actions (or the expectations about the actions) of players in period t onwards. In this environment, he defines a subgame-perfect equilibrium to be symmetric if its continuation value is independent of the past history of play. A reconsideration-proof equilibrium is then an equilibrium that achieves the highest payoffs within symmetric equilibria.

[7] Kocherlakota defines a "state" in his work, but this state only depends on the expectation about current and future actions, which makes it really not a state. In our case, we define a state as arising from past actions (including possibly past actions of nature, if randomness is present). This is in line with the literature on optimal control and dynamic programming. Our analysis can be extended straightforwardly to situations in which expectations about current and future actions affect the current set of actions and payoffs, as it happens in hybrid environments where some elements of competitive-equilibrium behavior coexist with strategic interactions.

The following proposition shows that an organizational equilibrium is reconsideration proof in the absence of state variables.

Proposition 2. *Consider a game where time-t preferences are given by*

$$V(a_t, a_{t+1}, a_{t+2}, \ldots)$$

and Assumption 2 holds. If a path $(a_0, a_1, ...)$ is an organizational equilibrium, then it is the outcome of a reconsideration-proof equilibrium.

Proof. First consider the set of sequences $\{a_\tau\}_{\tau=0}^{\infty}$ that satisfy the condition that lifetime utility is weakly increasing (condition 1 in Definition 2). Any sequence $\{a_\tau\}_{\tau=0}^{\infty}$ that attains the maximum of $V(a_0, a_1, \ldots)$ within this set has to be such that $V(a_t, a_{t+1}, \ldots) = \bar{V}$ for any t. If not, then there exists t such that $V(a_t, a_{t+1}, \ldots) < V(a_{t+1}, a_{t+2}, \ldots)$. The initial proposer can copy the sequence starting from agent in period $t+1$ by proposing $\{\widehat{a}_\tau\}$ where $\widehat{a}_\tau = a_{\tau+t+1}$. Next, consider the strategy proposed above, in which any deviation from the prescribed play leads to a restart of the path. According to this strategy, regardless of the past history, the path of play induced by the strategy going forward is a sequence $(a_s, a_{s+1}, ...)$ for some s.[8] The utility from any of these sequences to the player called to move is $\bar{V}$, which proves that the equilibrium is symmetric. By the definition of an organizational equilibrium, there exists no other sequence that could offer a constant value $\hat{V} > \bar{V}$ to all players. Since a reconsideration-proof equilibrium must give the same payoff to all players, it follows that the organizational equilibrium attains the maximum payoff among symmetric equilibria, which completes the proof. □

In the presence of a state variable, an organizational equilibrium assumes that players coordinate on strategies which only depend on the history of play h^t and not on the physical state. In this case, an organizational equilibrium imposes symmetry only in that the payoff of the subutility V is independent of the history of play, but the payoff of each time-t player is still different across

[8] If the player called to make decisions is player t, we have $s = t$ on the equilibrium path and $s < t$ if deviations occurred in the past.

histories which lead to different levels of the state. Intuitively, a different state implies a different set of possible utility levels going forward, so we should expect it to affect payoffs in the subgames going forward. However, this dependence of utility from the state takes a simple form under weak separability, and there is a natural mapping across histories with different levels of capital: the same sequences of actions are possible under any level of capital, and the preferences of player t over the sequences from date t on are also represented by the subutility V, independent of k_t. For this reason, imposing reconsideration proofness on preferences represented by V alone is appealing.

Our construct cannot of course completely get around the presence of a physical state, and it is for this reason that the no-delaying condition has some bite. In a reconsideration-proof equilibrium, where the state is not there, at any time t both the current and the *future* players' equilibrium payoffs are independent of the past history of play. This is no longer true in an organizational equilibrium when the state matters: while the current player receives the same equilibrium payoff for all histories that share the same state, future players do not.

To illustrate this point concretely, consider the example of Section 2. At each point t, the equilibrium payoff for player t depends on the past history of play only through k_t and not through the entire past history of actions; moreover, the equilibrium is such that player t is indifferent between all saving rates in $\left[q^*\left(\frac{\alpha\delta\beta}{1-\alpha\beta+\alpha\delta\beta}\right), s^*\right]$. However, *future* players would strictly prefer the equilibrium path that would unfold if player t chose s^*, which would lead to s^* being played for ever. If we appealed to (strong) Pareto optimality to select among equilibria, then s^* would be selected. But this equilibrium is suspect for the same logic that leads us to discard the trigger strategies that support the best subgame-perfect equilibrium for player t. Specifically, if player t anticipates that, as of $t+1$, players will coordinate on the Pareto-optimal equilbrium and will thus play s^* independently of past history, she has an incentive to play the best one-shot saving rate instead. Our no-delaying condition imposes that, whatever coordination mechanism selects the equilibrium to be played from period 0, no player at any time could be better off by deviating and counting on other players to use the same coordination mechanism to restart the game. Formally, given an equilibrium strategy profile σ, we require

$$V(\sigma_t(h^t), a_{t+1,\sigma|_{h^t}}, a_{t+2,\sigma|_{h^t}}, ...) \geq V(\tilde{a}_t, \sigma_0, a_{1,\sigma}, a_{2,\sigma}, ...) \quad \forall \tilde{a}_t \in A, h^t,$$

where we used the following short-hand notation:

$$a_{t+1,\sigma|_{h^t}} := \sigma_{t+1}(h^t, \sigma(h^t)),$$

$$a_{t+s,\sigma|_{h^t}} := \sigma_{t+s}(h^t, a_{t+1,\sigma|_{h^t}}, ..., a_{t+s-1,\sigma|_{h^t}}),$$

$$a_{1,\sigma} := \sigma_1(\sigma_0),$$

$$a_{s,\sigma} := \sigma_s(\sigma_0, a_{1,\sigma}, ...a_{s-1,\sigma}).$$

It's useful to compare organizational equilibrium to two alternative notions of equilibrium for dynamic games which are inspired by similar concerns about what constitutes a "credible punishment" in subgame-perfect equilibria.

An extension of reconsideration-proofness to environments with state variables was already proposed by Nozawa (2014). Nozawa requires weakly reconsideration proof equilibria to be such that the equilibria of all subgames share the same payoff *function* $\Psi(k)$, which depends on the state; in the absence of the state, this reduces to Kocherlakota's (1996) symmetry requirement. A strong reconsideration-proof equilibrium is then an equilibrium in which $\Psi(k)$ is undominated by any other equilibrium *point by point*. This is often too strong a requirement, and hence existence may fail. As an example, no reconsideration-proof equilibrium would exist in the example of Section 2.

An alternative approach is revision proofness, which was introduced by Asheim (1997) and made explicit as a game in Ales and Sleet (2014). In their papers, a larger class of credible punishments is allowable. Specifically, under reconsideration proofness, if Σ is the set of equilibrium strategies of the game, each player at any time t is allowed to coordinate current and future play to her favorite element of Σ. Under revision proofness, player t's coordination power is limited because she is required to propose deviations from the equilibrium path of play that benefit *all* future players. The resulting equilibrium set is much larger. For the case of quasi-geometric discounting with linear preferences, Ales and Sleet (2014) show that all subgame-perfect paths better than the Markov equilibrium are revision proof. In environments with state variables, a limitation of revision proofness is that it is unclear how a future player could "block" a revision proposal when she would inherit a different state under the revision proposal and would thus not be able to continue with the original strategy.

Our notion of organizational equilibrium retains the unilateral aspect of deviations from reconsideration proofness, but it relies on weak separability to define and impose symmetry across different levels of capital. The role of Pareto optimality enters in a limited way through the no-restarting condition and potentially through a final selection of a Pareto optimal path among those that satisfy symmetry and no-restarting.

3.2 Existence

To prove existence of an organizational equilibrium, we proceed in two steps. First, we rely on Kocherlakota (1996) to prove the existence of a reconsideration-proof equilibrium for the game with payoff function $V(.)$. We then use Assumption 2 to argue that the threat of restarting from the period-0 path after a deviation is a sufficient deterrent, no matter which deviation a player is considering.

The following assumptions mirror Kocherlakota's:

Assumption 3.

1. *A is a convex compact subset of a locally convex topological linear space with topology ρ_x.*
2. *V is quasiconcave over A^∞.*
3. *V is continuous over A^∞ with respect to the product topology ρ_x^∞.*

Under Assumption 3, Proposition 4 in Kocherlakota (1996) proves that a reconsideration-proof equilibrium exists for the game whose period-t payoff is $V(a_t, a_{t+1}, a_{t+2}, ...)$. This equilibrium achieves the maximal utility within symmetric equilibria.

Proposition 3. *Under Assumptions 2 and 3, an organizational equilibrium exists.*

Proof. Let $(a_0^E, a_1^E, \ldots)$ be the outcome of a reconsideration-proof equilibrium for the game whose period-t payoff is $V(a_t, a_{t+1}, a_{t+2}, ...)$, and let $\bar{V}$ be its associated value. This means that, for any period t and any actions $a \in A$, there exists a continuation sequence $(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)$ which is also a reconsideration-proof equilibrium and is such that

$$V(a_t^E, a_{t+1}^E, a_{t+2}^E, \ldots) \geq V(a, a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots).$$

We then have

$$V(a, a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots) = \widetilde{V}(a, \widehat{V}(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)).$$

Acknowledging that the sequence $(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots)$ is potentially a function of the deviation a (as well as of time t, which we can hold fixed), define

$$\underline{V} := \inf_{a \in A} \widehat{V}(a_{t+1}^{\bar{E}}, a_{t+2}^{\bar{E}}, \ldots). \tag{3.5}$$

By the compactness of A, Tychonoff's theorem, and continuity of $\widehat{V}$, we can find a sequence of actions $a_0^*, a_1^*, ...$ that attains the infimum in equation (3.5) above. Exploiting Assumption 2, this sequence ensures subgame perfection and satisfies the no-restarting condition:

$$V(a_0^*, a_1^*, a_2^*, \ldots) \geq V(a, a_0^*, a_1^*, \ldots).$$

This path attains a value $\bar{V}$. Since the set of organizationally admissible sequences is contained in the set of outcomes of symmetric equilibria, it follows that this path attains the highest payoff among organizationally admissible sequences and is therefore an organizational equilibrium for the game without state variables. By weak separability (Assumption 1), this property carries over to the original game with state variables. Hence, playing $(a_0^*, a_1^*, ...)$ followed by a restart after any deviation is an organizational equilibrium. □

We conclude this section by studying the case in which the function $\hat{V}$ admits a recursive structure, as is the case in the example of Section 2 and in many other applications of economic interest. This structure in turn provides a useful way to compute and characterize organizational equilibria.

Assumption 4. *There exists a function $W : A \times \mathbb{R} \to \mathbb{R}$, increasing in the second argument, such that, given any sequence $\{a_t\}_{t=0}^{\infty} \in A^{\infty}$,*

$$\widehat{V}(a_0, a_1, a_2, \ldots) \equiv W\left(a_0, \widehat{V}(a_1, a_2, \ldots)\right), \tag{3.6}$$

Proposition 4. *Under Assumptions 2, 3 and 4, there exists an organizational equilibrium $\{a_t\}_{t=0}^{\infty}$ which is recursive in the value $\widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$: that is, there exists a function $g : \mathbb{R} \to A \times \mathbb{R}$ such that $(a_t, v_{t+1}) = g(v_t)$, and $v_t = \widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$ for all $t = 0, 1, \ldots$.*

Proof. See Appendix B. □

Proposition 4 uses values as a state variable in ways similar to Abreu, Pearce, and Stacchetti (1986, 1990). However, as the proof shows, constructing the set of possible values is considerably more involved than in the case of Abreu, Pearce, and Stacchetti; hence, while the proposition greatly simplifies the task of constructing organizational equilibria, it still leaves a challenging task. To make further progress, we need one more assumption:

[to be completed]

Proposition 5. *Under Assumptions 2, 3, and 4, the value of $\widehat{V}(a_t, a_{t+1}, a_{t+2}, \ldots)$ is increasing over time for any organizational equilibrium, and it converges to the value associated with the steady state which maximizes $V(a, a, a, \ldots)$. Furthermore, if $\widehat{V}$ is a strictly quasiconcave function and the steady state that maximizes $V(a, a, a, \ldots)$ is not a Markov equilibrium. Then the initial value of $\widehat{V}(a_0, a_1, a_2, \ldots)$ is strictly below the steady state: convergence is not immediate.*

Proof. See Appendix B □

Proposition 5 provides a way of characterizing organizational equilibria. We first compute a steady state that maximizes $V(a, a, a, ...)$: this is the steady state that would be chosen by the decision maker at time 0 if she could commit future players to take the same action. This maximization yield a value V^* which must remain constant along the path, i.e., $V(a_t, a_{t+1}, a_{t+2}, ...) = V^*$. We then construct a path that leads to this steady state, using the no-delay condition to inform us of the starting point. The last part of the proof shows that convergence to the steady state takes time, unless we are in a special case in which the steady state can be supported in a Markov equilibrium with no intertemporal incentives.

4 Approximated Equilibrium

In this section, we first introduce a class of weakly separable economies with the quasi-geometric discounting preferences. This class include the example discussed in Section 2 as a special case, and it also include some interesting models explored in the literature. In general, weakly separable is a quite demanding requirement. We then proceed to provide an approximation algorithm for models that are not weakly separable.

4.1 A Class of Weakly Separable Economies

Consider the following class of economies. The state variable is k and the action is a. The preference is specified as

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} \, u\left(k_{t+\tau}, a_{t+\tau}\right).$$

Suppose that the period utility function takes the form of

$$u(k, a) = C_{10} + C_{11} h(k) + C_{12} m(a),$$

and the state evolves according to

$$h(k') = C_{20} + C_{21}h(k) + C_{22}g(a),$$

for some monotonic functions h, g, and m, and some constant matrix $\mathbf{C}$. Note that in Section 2, the economy is corresponding to $h(k) = \log(k)$, $g(a) = \log(a)$ and $m(a) = \log(1-a)$. It is easy to verify that this class of economies are weakly separable. To see this, given a sequence of actions $\{a_\tau\}_{\tau=0}^{\infty}$ and the initial state k_0, the sequence of state variables follows

$$h(k_t) = C_{20}\frac{1-C_{21}^t}{1-C_{21}} + C_{21}^t h(k_0) + C_{22}\sum_{\tau=0}^{t-1} C_{21}^{t-1-\tau} g(a_\tau),$$

and the lifetime utility is given by

$$\begin{aligned} U(k_0, \{a_\tau\}_{\tau=0}^{\infty}) = &\frac{1-\beta+\delta\beta}{1-\beta}C_{10} + \frac{C_{11}(1-\beta C_{21}+\delta\beta C_{21})}{1-\beta C_{21}}h(k_0) \\ &+ C_{12}m(a_0) + \frac{C_{11}C_{22}\delta\beta}{1-\beta C_{21}}g(a_0) + \delta\sum_{j=1}^{\infty}\beta^j\left(C_{12}m(a_j) + \frac{\beta C_{11}C_{22}}{1-\beta C_{21}}g(a_j)\right). \end{aligned}$$

We now turn to describe two examples that belong to this weakly separable class. Both of them are widely used in the literature.

Linear production function and CRRA utility function This example can be interpreted as a growth model with a linear production or a consumption-saving problem with a constant interest rate. Without the loss of generality, the resource constraint can be written as $c + k' = \theta k$. Assume the period utility function takes the following CRRA form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and the lifetime utility for an agent in period t is

$$\Psi_t = u(c_t) + \delta\sum_{\tau=1}^{\infty}\beta^\tau u\left(c_{t+\tau}\right).$$

We use the saving rate as the rescaled action, and show that Assumption 1 is satisfied. Given an initial capital level k_0 and a sequence of saving rate $\{s_\tau\}_{\tau=0}^{\infty}$, the implied sequence of capital is simply

$k_t = \Pi_{\tau=0}^{t-1} s_\tau \theta^t k$. The lifetime utility can be rewritten as

$$U(k_0, \{s_\tau\}_{\tau=0}^{\infty}) = \frac{(\theta k_0)^{1-\sigma}}{1-\sigma} \Big\{ (1-s_0)^{1-\sigma} + \delta\beta(s_0(1-s_1)\theta)^{1-\sigma} + \delta\beta^2(s_0 s_1(1-s_2)\theta^2)^{1-\sigma} + \dots \Big\}$$
$$\equiv \frac{(\theta k_0)^{1-\sigma}}{1-\sigma} V(\{s_\tau\}_{\tau=0}^{\infty}),$$

which is weakly separable. Therefore, we can apply our organizational equilibrium concept in this environment.

Leisure choice Consider now the case where agents also choose the amount of labor to supply. The production function now includes labor as input $f(k_t, \ell_t) = k_t^\alpha \ell_t^{1-\alpha}$, and the resource constraint is $c_t + k_{t+1} = f(k_t, \ell_t)$. Assume the period utility function is $u(c_t, \ell_t) = \log c_t + \frac{(1-\ell_t)^{1-\gamma}}{1-\gamma}$, and the lifetime utility for the agent at period t is given by

$$\Psi_t = u(c_t, \ell_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u(c_{t+\tau}, \ell_{t+\tau}).$$

We choose saving rate $s \in [0, 1]$ and labor $\ell \in [0, 1]$ as the rescaled action. With initial capital $k_0 = k$, the saving rate sequence $\{s_\tau\}_{\tau=0}^{\infty}$, and the labor sequence $\{\ell_\tau\}_{\tau=0}^{\infty}$. This will imply the sequence of capital as $k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-1-j}} \ell_j^{(1-\alpha)\alpha^{t-1-j}}$. The total payoff is

$$\begin{aligned} & U(k_0, s_0, s_1, \dots, \ell_0, \ell_1, \dots) \\ = & \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_0 + \log(1-s_0) + \frac{\delta\alpha\beta}{1-\alpha\beta} \log s_0 + \delta \sum_{j=1}^{\infty} \beta^j \left(\log(1-s_j) + \frac{\alpha\beta}{1-\alpha\beta} \log s_j \right) \\ & + \frac{(1-\ell_0)^{1-\gamma}}{1-\gamma} + \frac{\delta\alpha(1-\alpha)\beta}{1-\alpha\beta} \log \ell_0 + \delta \sum_{j=1}^{\infty} \beta^j \left(\frac{(1-\ell_j)^{1-\gamma}}{1-\gamma} + \frac{\alpha(1-\alpha)\beta}{1-\alpha\beta} \log \ell_j \right). \end{aligned}$$

The weakly separable requirement is satisfied by both the saving rates and the leisure choices, and we can apply our equilibrium concept to each choice separately.

4.2 Approximation with Weakly Separable Economies

The assumption of weakly separable utility is quite restrictive and is often not satisfied. In this section, we propose a strategy to study organizational equilibrium for economies where such assumption is not satisfied. Our approach is to look at an economy that is weakly separable and very similar in a particular metric to the original one, and then study organizational equilibrium in this alternative economy. This strategy has a strong tradition in Macroeconomics where little (if anything) is known about recursive equilibrium in distorted economies that do not have a particular functional form. Consequently, the equilibrium is computed for a similar economy in a certain sense (See Kubler and Schmedders (2005) and Kubler (2007) for a discussion).

To illustrate our point, consider a quasi-geometric discounting economy with state variable k and action a. Suppose the preference is

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(k_{t+\tau}, a_{t+\tau}\right),$$

and the state variable evolves according to

$$k_{t+1} = F(k_t, a_t).$$

In general, Ψ_t may not be separable between k and the sequence of actions $\{a_{t+\tau}\}_{\tau=0}^{\infty}$. Instead, we consider the log-linearized version of the original problem around a particular point $(\overline{k}, \overline{a})$. This approximated economy is

$$\widehat{\Psi}_t = \widehat{u}(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau}\ \widehat{u}\left(k_{t+\tau}, a_{t+\tau}\right).$$

such that

$$\widehat{u}(k, a) = u\left(\overline{k}, \overline{a}\right) + \overline{k} u_k\left(\overline{k}, \overline{a}\right)(\log(k) - \log(\overline{k})) + \overline{a} u_a\left(\overline{k}, \overline{a}\right)(\log(a) - \log(\overline{a}))$$

$$\log(k') = \log\left(\overline{k}\right) + F_k\left(\overline{k}, \overline{a}\right)(\log(k) - \log(\overline{k})) + \frac{\overline{a} F_a\left(\overline{k}, \overline{a}\right)}{\overline{k}}(\log(a) - \log(\overline{a}))$$

This approximated economy belongs to the class of separable economies discussed in Section 4.1, and therefore it is weakly separable. Unlike in a standard log-linearization exercise, the stationary allocation of this economy under the organizational equilibrium is not known ex ante. Denote the converging point of the transition path $\{a_\tau\}_{\tau=0}^{\infty}$ in the organizational equilibrium as a^*. The natural requirements for the selection of $(\overline{k}, \overline{a})$ are $a^* = \overline{a}$ and $\overline{k} = F(\overline{k}, \overline{a})$. These lead to the following condition that characterizes the steady state $\overline{a}$

$$(1-\beta(1-\delta))\left(1-\beta F_k\left(\overline{k},\overline{a}\right)\right)u_a\left(\overline{k},\overline{a}\right)+\delta\beta u_k\left(\overline{k},\overline{a}\right)F_a\left(\overline{k},\overline{a}\right)=0. \tag{4.1}$$

Once $(\overline{k}, \overline{a})$ are fixed according to equation (4.1), the entire transition path can be derived similar to Section 2.2

$$a_{t+1} = exp\left\{\frac{\log(a_t)+\frac{\delta\beta u_k(\overline{k},\overline{a})F_a(\overline{k},\overline{a})}{u_a(\overline{k},\overline{a})(1-\beta F_k(\overline{k},\overline{a}))}\log(a_t)-(1-\beta(1-\delta))\log(\overline{a})}{\beta(1-\delta)}\right\}. \tag{4.2}$$

The detailed derivation can be found in the Appendix. We will also utilize this approximated economy in the quantitative taxation problem in the next section.

5 Organizational Equilibrium and Public Policy

In this Section, we put the notion of Organizational Equilibrium to work for the cases that we find most interesting, those of the determination of government policies when the Ramsey solution is time inconsistent.

To look at these environments we extend the framework in Section 3 to accommodate a government (a large player) who behaves strategically, and representative households who behave competitively. Given the current level of $k \in K$, the government chooses an action a from a set A, and the consumers choose an action s from the set $s(k) \subseteq S$. The state evolves according to $k' = F(k, x, s)$. Let the preferences for the government in period t are given by $\Psi(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots)$.[9]

[9] As in Section 3, sometimes it may be necessary to transform the original government action so that it is feasible independently of the choices of the private sector and the current level of the physical state, and so that the desired

Assumption 5. *Given a sequence of government actions* $\boldsymbol{a} := \{a_t\}_{t=0}^{\infty}$*, there exists a unique competitive equilibrium* $\boldsymbol{s(a)} := \{s_t(\boldsymbol{a})\}_{t=0}^{\infty}$*, where the sequence* $\boldsymbol{s(a)}$ *is independent of the state* k_0*.*

Assumption 5 plays two roles. First, the uniqueness allows us to define government preferences directly over the sequence of government actions, taking as given that households will play the associated competitive equilibrium. Second, the fact that $\mathbf{s}$ is independent of the initial state extends the weak separability requirement that is at the heart of our method. We can then define the government's preferences over sequences of actions as

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) := \Psi(k, a_t, s_t(\mathbf{a}), a_{t+1}, s_{t+1}(\mathbf{a}), a_{t+2}, s_{t+2}(\mathbf{a}), \ldots), \tag{5.1}$$

where for $t \geq 0$, k_t is computed recursively as

$$k_{t+1}(k) = F(k_t(k), x_t, s_t(\mathbf{a})). \tag{5.2}$$

These preferences take now the same form as in the one-agent case, so we impose once again Assumptions 1 and 2, and we define an organizational equilibrium as in Definition 1. While the definition of an organizational equilibrium is the same in terms of sequences of actions, its connection to symmetric subgame-perfect equilibria of an underlying game is slightly different, due to the presence of competitive households that act in anticipation of the government's future actions. We describe this game in detail in Appendix D.1. Two aspects are worth pointing out. First, as in the application that we will describe shortly, we assume that the government is a first mover within each period, so that households react contemporaneously to a government deviation.[10] Second, the equilibrium strategies that gradually reward the government from abstaining from short-run temptations and conversely reverse those rewards in the event of a deviation rely on a coordination of the beliefs of the private sector, rather than simply on the actions of future policymakers.

separability property of preferences emerges. A similar rescaling may be needed for the household choices.

[10]Of course, the definition could be adapted to environments where the opposite timing prevails.

5.1 A Simple Taxation Example

To illustrate the general definition of an organizational equilibrium in a hybrid competitive-strategic environment, we revisit Klein, Krusell, and Ríos-Rull (2008), replacing their Markov equilibrium with our notion of organizational equilibrium. In this problem, the government sets a tax instrument, which, depending on the case, is a flat tax on capital income, labor income, or total income. The proceeds are used to produce a public good, and the government is constrained to a balanced budget. In this subsection, we first consider a special case with inelastic labor supply and full depreciation, where closed-form solution is possible. We then explore the quantitative version as in Klein, Krusell, and Ríos-Rull (2008) in the next subsection.

The production function is given by

$$y_t = f(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha},$$

where labor is inelastically supplied ($\ell_t = 1$) and capital is subject to full depreciation, so that the resource constraint is

$$c_t + g_t + k_{t+1} = f(k_t, \ell_t). \tag{5.3}$$

g_t is the government provision of the public good. Preferences are

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_t + \gamma \log g_t\right].$$

We derive the analytical expressions for the case in which the government instrument is a tax on capital income. The same method can be applied when the tax is levied on labor income, or on total income. The consumers' budget constraint is

$$c_t + k_{t+1} = (1 - \tau_t) r_t k_t + w_t.$$

We take the tax rate to be the government's action. Its domain is $[0, 1]$ and is thus independent of initial capital. To avoid dealing with the complications of infinitely negative utility, we constrain the government to choices in $[\epsilon, 1-\epsilon]$, where $\epsilon > 0$ can be chosen arbitrarily small so that the bounds are never hit in the equilibrium we consider. Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$, we first characterize a competitive equilibrium in terms of sequences of consumption, capital, and factor prices, and then summarize it by a sequence of saving rates $s_t \in [0, 1]$, which is our notion of private sector's actions.

Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ and an initial level of capital k_0, a sequence $\{c_t, g_t, k_{t+1}, w_t, r_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if the following conditions are satisfied:

- Factor prices are equal to their marginal productivity, i.e.,

$$r_t = f_k(k_t),$$
$$w_t = f(k_t) - r_t k_t;$$

- The household's intertemporal decision is optimal, which requires the Euler condition to hold

$$u'(c_t) = \beta u'(c_{t+1})(1 - \tau_{t+1}) r_{t+1},$$

  along with the transversality condition

$$\lim_{t \to \infty} \beta^t u'(c_t) k_{t+1} = 0;$$

- The government budget is balanced, i.e.,

$$g_t = \tau_t r_t k_t;$$

- And the resource constraint (5.3) holds.

Substituting factor prices, the resource constraint, and the budget constraint into the Euler equation and summarizing private-sector actions by the saving rage $s_t := k_{t+1}/f(k_t, \ell_t)$ (which also is in $[0, 1]$

independently of initial capital), a competitive equilibrium is described by the difference equation

$$\frac{s_t}{1-s_t-\alpha\tau_t} = \frac{\alpha\beta(1-\tau_{t+1})}{1-s_{t+1}-\alpha\tau_{t+1}}. \tag{5.4}$$

along with the transversality condition

$$\lim_{t\to\infty} \beta^t \frac{s_t}{1-\alpha\tau_t-s_t} = 0. \tag{5.5}$$

Lemma 1. *Assumption 5 is satisfied for this economy. Specifically, given a sequence* $\{\tau_t\}_{t=0}^{\infty} \in [\epsilon, 1-\epsilon]^{\infty}$*, there exists a unique competitive equilibrium.*

Suppose the sequence of tax rates is $\{\tau_j\}_{j=0}^{\infty}$, the sequence of saving rates is $\{s_\tau\}_{\tau=0}^{\infty}$, and the initial capital is k_0. Then the sequence of capitals is,

$$k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-1-j}},$$

and the current government's total payoff is

$$U(k_0, s_0, s_1, \ldots, \tau_0, \tau_1, \ldots) = \frac{\gamma}{1-\beta}\log\alpha + \frac{\alpha(1+\gamma)}{1-\alpha\beta}\log k_0 + \sum_{j=0}^{\infty}\beta^j\left\{\log(1-\alpha\tau_j-s_j)+\gamma\log\tau_j+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_j\right\}.$$

Clearly, the weakly separable condition is satisfied in this environment. In an organizational equilibrium, the action payoff should be equalized for governments in different periods, i.e., $\sum_{j=0}^{\infty}\beta^j\left\{\log(1-\alpha\tau_{t+j}-s_{t+j})+\gamma\log\tau_{t+j}+\frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s_{t+j}\right\}$ should equal to a constant for different t. Utilizing the recursive structure, the following lemma formalizes this idea.

Lemma 2. *In an organizational equilibrium, given the current tax rate* τ*, then the current saving rate* s*, the future tax rate* τ' *and* s' *need to satisfy the following system of equations for some constant*

$\overline{V}$

$$\overline{V} = \log(1-\alpha\tau - s) + \gamma\log\tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s, \tag{5.6}$$

$$\overline{V} = \log(1-\alpha\tau' - s') + \gamma\log\tau' + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s', \tag{5.7}$$

$$\frac{1-s'-\alpha\tau'}{1-\tau'} = \frac{\alpha\beta(1-s-\alpha\tau)}{s}. \tag{5.8}$$

Intuitively, equation (5.6) and (5.7) make sure that the action payoff for the current and future government are the same. Equation (5.8) is corresponding to the Euler equation in the private sector. Given τ, there could be two different saving rates s that satisfy equation (5.6). To proceed, define $h\left(\tau;\overline{V}\right)$ as

$$h\left(\tau;\overline{V}\right) = \min\left\{ s\in(0,1) \;\middle|\; \log(1-\alpha\tau-s) + \gamma\log\tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log s = \overline{V} \right\} \tag{5.9}$$

which selects the smaller saving rates that delivers the action payoff $\overline{V}$. The system (5.6) to (5.8) is too complicated to allow analytical solutions, and we have verified numerically that there does not exist a solution to the system if the larger saving rate in equation (5.6) is selected. Therefore, in terms of equation (5.7), only $h(\tau';\overline{V})$ can be chosen as well. Otherwise, there will be no solution in the next period. This leads to the following proposition that characterizes the organizational equilibrium.

Proposition 6. *The sequence of tax rates in the organizational equilibrium can be obtained recursively by the proposal function $q(\tau)$ which satisfies*

$$\frac{1-h(q(\tau);V^*)-\alpha q(\tau)}{1-q(\tau)} = \frac{\alpha\beta(1-h(\tau;V^*)-\alpha\tau)}{h(\tau;V^*)}, \tag{5.10}$$

where V^* is defined as

$$V^* = \max_{\tau}\ \log(1-\alpha\tau-\alpha\beta(1-\tau)) + \gamma\log\tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta}\log(\alpha\beta(1-\tau)). \tag{5.11}$$

The initial tax rate τ_0 is chosen such that $\Phi(\tau_0) \leq V^*$, where $\Phi(\tau_0)$ is given by

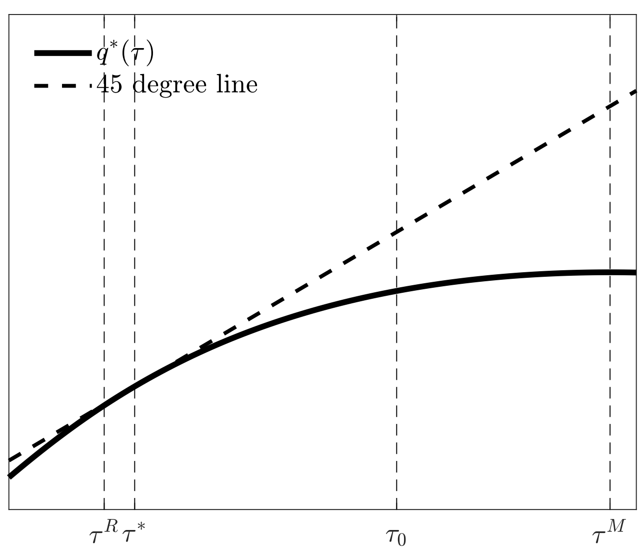
$$\Phi(\tau_0) = \max_{\tau_{-1}} \; \log(1 - \alpha\tau_{-1} - s_{-1}) + \gamma \log \tau_{-1} + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_{-1} \tag{5.12}$$

subject to

$$\frac{1 - s_1(\tau_0) - \alpha\tau_0}{1-\tau_0} = \frac{\alpha\beta(1 - s_{-1} - \alpha\tau_{-1})}{s_{-1}}. \tag{5.13}$$

In equilibrium, the government in each period will obtain the same constant action payoff V^*. However, the no-waiting condition prevents a constant tax rate, and the sequence of tax rates only approach to its steady state level τ^* gradually. If the initial tax rate τ_0 is known, then the entire transition path can be computed recursively via the proposal function $q(\tau)$. The condition that $\Phi(\tau_0) \leq V^*$ then guarantees that the initial government has no incentive to wait for the next government to make the equilibrium proposal. As in Section 2.2, we will select τ_0 such that $\Phi(\tau_0) = V^*$.

FIGURE 6: Proposal Function $q^*(s)$



In the appendix, we describe the details of the Markov equilibrium and the Ramsey outcome. Let

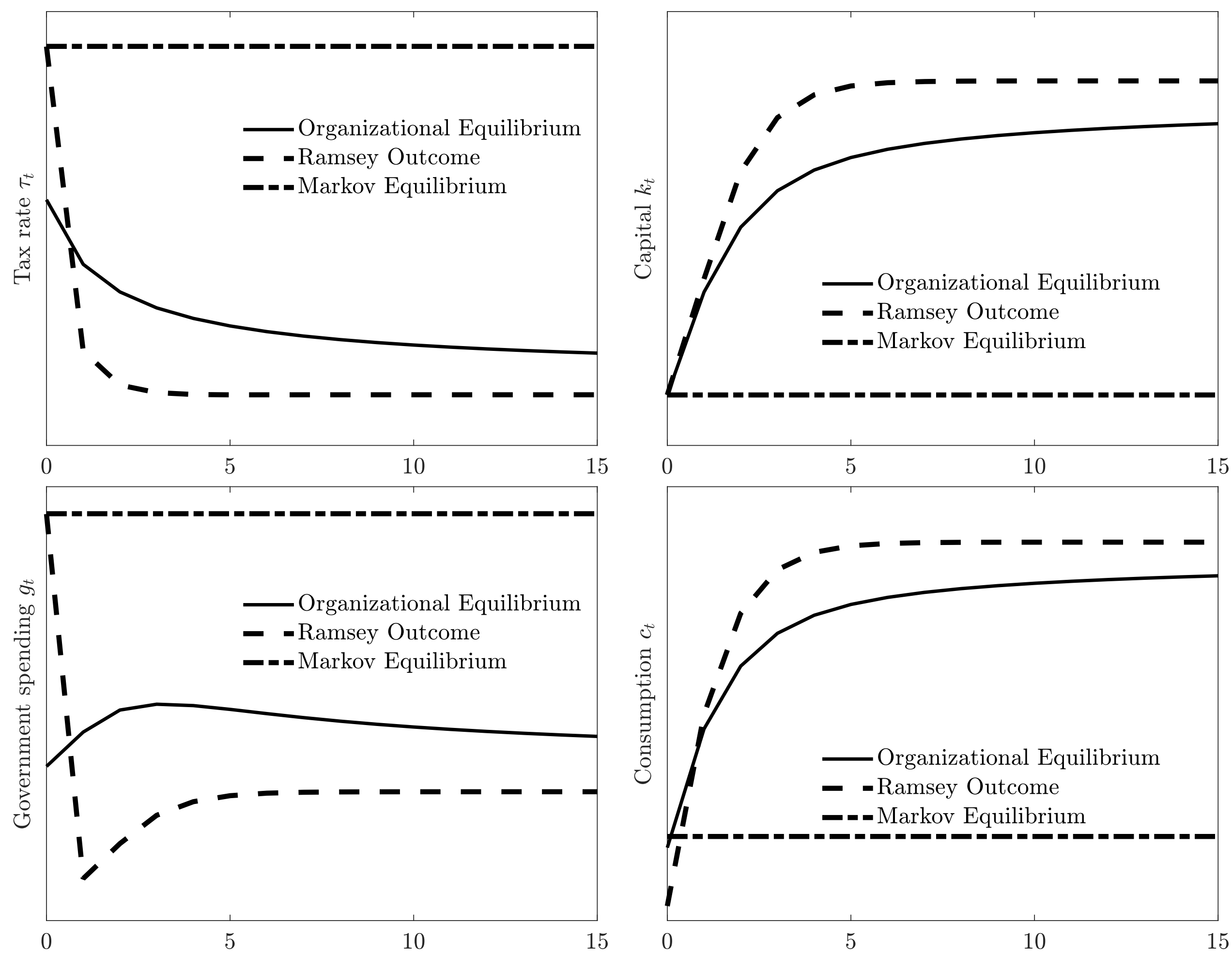
τ^M denotes the tax rate in the Markov equilibrium and τ^R denote the steady state tax rate in the Ramsey outcome. The In the Markov economy, the government tends to choose a high tax rate because they fail to take into account the effects of the current tax rate on past saving choices. While in the Ramsey outcome, the government internalizes this effect and will set a lower tax rate in the long run. In the organizational equilibrium, the proposal function $q(\tau)$ governs the dynamics of the tax sequence. As a numerical example, we set $\beta = 0.9, \alpha = 0.36$, and $\gamma = 0.5$, and the proposal function is plotted in Figure 6. As expected, the initial tax rate τ_0 is lower than τ^M, but it is higher than τ^R. The proposal function implies a gradual transition of the tax rate from τ_0 to the steady state τ^*.

Figure 7 displays the corresponding transition paths for the tax rates and allocation in the three economies. The initial capital is chosen to be the steady state capital level in the Markov economy. In the Ramsey outcome, the government initially sets the tax rate as high as in the Markov equilibrium, and rapidly adjusts it to the steady state value τ^R. As a result, the private consumption drops initially since households anticipating a lower tax rate in the future. At the same time, the capital stock accumulates to its steady state high level. The path of government spending is non-monotonic, since the output and tax rate move in the opposite direction. In the organizational equilibrium, the tax rates starts lower than the Markov tax rate, and it converges to τ^* which is still higher than the long-run level in the Ramsey outcome. The transition of the tax rate is slower than the Ramsey outcome to make sure that the government action payoff is equalized across periods. Because of a lower tax rate, the capital stock is higher than the Markov equilibrium.

5.2 A Quantitative Taxation Model

In this section, we revisit the quantitative taxation problem in Klein, Krusell, and Ríos-Rull (2008). We extend previous section to allow elastic labor supply, partial depreciation, and three types of

Figure 7: Transition Path



taxation. The preference is

$$\sum_{t=0}^{\infty} \beta^t \left[\gamma_c \log c_t + \gamma_\ell \log(1 - \ell_t) + \gamma_g \log g_t\right].$$

where ℓ_t stands for labor. The budget constraint for the household is

$$c_t + i_t = w_t \ell_t + r_t k_t - (\tau_t^\ell + \tau_t) w_t \ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k + \tau_t)}{r_t}\right) r_t k_t$$

where i_t is investment, τ_t^ℓ is labor income tax, τ_t^k is capital income tax, and τ_t is total income tax. The last term on the right-hand of the budget constraint allows for the possibility of capital depreciation deduction. In Klein, Krusell, and Ríos-Rull (2008), the capital evolves according to

$$k_{t+1} = (1-\delta)k_t + i_t.$$

However, this specification does not allow the government's lifetime utility to be separable between capital and the sequence of tax rates. Instead, we specify the law of motion of capital to be

$$\log k_{t+1} = \log \overline{k} + (1-\delta)\log k_t + \delta \log i_t.$$

This can be viewed as a log-linear approximation of the original law of motion, and it fits into the class of separable economies discussed in Section 4.1. This modification delivers the weakly separable property, and therefore we can apply the organizational equilibrium to this approximated economy.

We set the parameters to be $\alpha = 0.36, \beta = 0.96, \delta = 0.08, \gamma_g = 0.09, \gamma_c = 0.27, \gamma_\ell = 0.64$. We choose γ_g such that the steady state government spending to GDP ratio is 18% in the Pareto efficient allocation, and γ_ℓ such that the working time is 35% of a day. The rest of the parameters are standard in the literature. The properties of the transition path are very similar to what we have shown in the previous section, and in this part we discuss mainly the tax rates and allocation in the steady state. Table 1 shows the steady state comparison among the Pareto allocation, the Ramsey outcome, the Markov equilibrium, and the organizational equilibrium. The results regarding the Ramsey outcome and the Markov equilibrium are very similar to those obtained in Klein, Krusell, and Ríos-Rull (2008), and we will focus our attention to the organizational equilibrium. Throughout the three different tax instruments, a common feature is that the tax rates and the allocation in the organizational equilibrium always stay between the Ramsey outcome and the Markov equilibrium. This feature should be well understood from the discussion in Section 2.2 and Section 5.1. The capital income taxation is the most distortionary one among the three taxes. In the Markov equilibrium, the output level is only 35% compared with the Pareto efficient output level, while the output level

Table 1: Steady State Comparison

Aggregate statistics	Labor income tax				Capital income tax				Total income tax			
	Pareto	Ramsey	Markov	Organization	Pareto	Ramsey	Markov	Organization	Pareto	Ramsey	Markov	Organization
y	1.000	0.701	0.711	0.706	1.000	0.588	0.347	0.553	1.000	0.669	0.679	0.674
k/y	2.959	2.959	2.959	2.959	2.959	1.735	0.624	1.529	2.959	2.528	2.579	2.553
c/y	0.510	0.510	0.544	0.527	0.510	0.712	0.666	0.704	0.510	0.533	0.555	0.544
g/y	0.254	0.254	0.219	0.236	0.254	0.149	0.284	0.174	0.254	0.265	0.239	0.252
c/g	2.008	2.008	2.483	2.234	2.008	4.785	2.345	4.045	2.008	2.008	2.325	2.156
ℓ	0.350	0.245	0.249	0.247	0.350	0.278	0.292	0.280	0.350	0.256	0.257	0.256
τ		0.397	0.342	0.369		0.673	0.916	0.732		0.332	0.301	0.317

in the organizational equilibrium is 55% of the Pareto allocation and it is only slightly below the Ramsey outcome. We interpret this result as a large improvement over the Markov equilibrium. For labor income tax and total income tax, the difference between the Ramsey outcome and the Markov equilibrium is much smaller. Since the organizational equilibrium stays in between of the two benchmarks, we only conclude that it brings the allocation closer to the Ramsey outcome.

6 Conclusion

TO BE ADDED.

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Appendix

A Sustainable Equilibrium in Section 2

The initial agent's problem is

$$\max_{\{c_t\}_{t=0}^{\infty}} \quad u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t u(c_t) \qquad \text{s.t.}$$
$$c_t + k_{t+1} = f(k_t),$$
$$u(c_t) + \delta \sum_{j=1}^{\infty} \beta^t u(c_{t+j}) \geq \Phi^M(k_t),$$
$$k_0 \text{ given .}$$

Let ξ_t and λ_t denote the multipliers associated with the resource constraint and the participation constraint. The Lagrangian is

$$J = (1-\delta)u(c_0) + \delta \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \lambda_t \left((1-\delta)u(c_t) + \delta \sum_{k=0}^{\infty} \beta^t u(c_{t+k}) - \phi^M(k_t) \right) - \xi_t \left(c_t + k_{t+1} - f(k_t)\right) \right\}.$$

Note that

$$\sum_{t=0}^{\infty} \beta^t \lambda_t \delta \sum_{k=0}^{\infty} \beta^t u(c_{t+k}) = \delta \sum_{t=0}^{\infty} \beta^t \mu_t u(c_t),$$

where

$$\mu_t = \mu_{t-1} + \lambda_t, \quad \text{with } \mu_{-1} = 0.$$

It follows that

$$J = (1-\delta)u(c_0) + \delta \sum_{t=0}^{\infty} \beta^t \left\{ (1 + \lambda_t + \delta\mu_{t-1})u(c_t) - \lambda_t \Phi^M(k_t) - \xi_t \left(c_t + k_{t+1} - f(k_t)\right) \right\}$$

Frot $t > 0$, the first order conditions with respect to c_t and k_{t+1} are

$$(1 + \lambda_t + \delta\mu_{t-1})u_c(c_t) = \xi_t,$$
$$\beta(\xi_{t+1} f_k(k_{t+1}) - \lambda_{t+1}\Phi_k^M(k_{t+1})) = \xi_t,$$

which leads to

$$(1+\lambda_t+\delta\mu_{t-1})u_c(t)=\beta((1+\lambda_{t+1}+\delta\mu_t)u_c(c_{t+1})f_k(k_{t+1})-\lambda_{t+1}\Phi_k^M(k_{t+1}))$$

and can be rewritten as

$$(1+\lambda_t+\delta\mu_{t-1})\frac{s_t}{1-s_t}=\beta\left((1+\lambda_{t+1}+\delta\mu_t)\frac{\alpha}{1-s_{t+1}}-\lambda_{t+1}\phi\right),$$

where $\phi=\frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}$. Define the normalized multipliers z_t and v_t as

$$z_t\equiv\frac{1+\delta\mu_t}{1+\lambda_t+\delta\mu_{t-1}},$$
$$v_{t+1}\equiv\frac{\lambda_{t+1}}{1+\lambda_t+\delta\mu_{t-1}}.$$

Then the allocation needs to satisfy

$$\frac{s_t}{1-s_t}=\beta\left((z_t+v_{t+1})\frac{\alpha}{1-s_{t+1}}-v_{t+1}\phi\right)$$
$$z_{t+1}=\frac{1+\delta\mu_{t+1}}{1+\lambda_{t+1}+\delta\mu_t}=\frac{z_t+\delta v_{t+1}}{z_t+v_{t+1}}$$

For $t=0$, we have

$$(1-\delta)u_c(0)+\delta(1+\lambda_0+\delta\mu_0)u_c(0)=\delta\xi_0$$
$$\beta(\xi_1 f_k(1)-\lambda_1 U_k(1))=\xi_0$$

which leads to

$$\left(\frac{1}{\delta}+\lambda_0+\delta\mu_0\right)\frac{s_0}{1-s_0}=\beta\left((1+\lambda_1+\delta\mu_0)\frac{\alpha}{1-s_1}-\lambda_1\phi\right).$$

From now on, we use a guess-and-verify approach to solve for the equilibrium saving rates. The first scenario is that the participation constraint does not bind in the steady state, i.e., $\mathcal{H}(s^R)>\mathcal{H}(s^M)$. If this is the case, then the participation constraint will never bind, and the sequence of saving rates in the Ramsey outcome will be the solution in the sustainable equilibrium. To verify this, one can simply set $\lambda_t=0$ for all $t\geq 0$. The first order conditions in period 0 and in subsequent periods are

$$\frac{1}{\delta}\frac{s^M}{1-s^M} = \beta\frac{\alpha}{1-s^R},$$
$$\frac{s^R}{1-s^R} = \beta\frac{\alpha}{1-s^R},$$

both of which are true.

The second scenario is that the participation constraint binds in the steady state, i.e., $\mathcal{H}(s^R) = \mathcal{H}(s^M)$. The conjecture is that the saving rate is $s_0 = s^M$ and $s_t = s^T$ for $t > 0$, where[11]

$$s^T = \max_s \left\{s : \mathcal{H}(s) = \mathcal{H}\left(s^M\right)\right\}.$$

In the steady state, the following system of equations need to be satisfied

$$z_\infty = \frac{z_\infty + \delta v_\infty}{z_\infty + v_\infty},$$
$$\frac{s^T}{1-s^T} = \beta\left((z_\infty + v_\infty)\frac{\alpha}{1-s^T} - \phi v_\infty\right).$$

The solution is that

$$z_\infty = \frac{s^T}{1-s^T}\frac{1-\alpha\beta}{\alpha\beta}.$$

To verify the conjecture, consider the first order condition in the first period where $\lambda_0 = \mu_0 = 0$,

$$\frac{1}{\delta}\frac{s^M}{1-s^M} = \frac{\alpha}{1-\alpha\beta} = \beta\left((1+\lambda_1)\frac{\alpha}{1-s^T} - \lambda_1\phi\right).$$

To make the first order condition hold, λ_1 has to equal to

$$\lambda_1 = \frac{v_\infty}{z_\infty}.$$

[11] It is easy to verify that both the two solutions satisfy the first order conditions, but the one with higher saving rate yields higher utility for the initial agent.

To see this more clearly, note that the steady state condition implies that

$$\frac{s^T}{1-s^T} = z_\infty \frac{\alpha}{1-\alpha\beta} = \beta\left((z_\infty + v_\infty)\frac{\alpha}{1-s^T} - \phi v_\infty\right),$$

which can be further written as

$$\frac{\alpha}{1-\alpha\beta} = \beta\left(\left(1+\frac{v_\infty}{z_\infty}\right)\frac{\alpha}{1-s^T} - \frac{v_\infty}{z_\infty}\phi\right) = \beta\left((1+\lambda_1)\frac{\alpha}{1-s^T} - \lambda_1\phi\right).$$

For $t \geq 1$, it is sufficient to show

$$z_1 = \frac{1+\delta\lambda_1}{1+\lambda_1} = \frac{1+\delta\frac{v_\infty}{z_\infty}}{1+\frac{v_\infty}{z_\infty}} = z_\infty.$$

B Proof of Proposition 4.

To prove this we rely on a useful lemma, which introduces a convenient way of representing equilibria through their values, similarly to Abreu, Pierce, and Stacchetti's (1986; 1990) method.[12]

Lemma 3. *Let $V^* \in \mathbb{R}$ and $\hat{\mathcal{V}} \subset \mathbb{R}$ be a value and a set of continuation values that satisfy the following properties:*

1. $$\forall a \in A \quad \exists \hat{v} \in \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) \leq V^*;$$

2. $$\forall v \in \hat{\mathcal{V}} \quad \exists (a, \hat{v}) \in A \times \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) = V^* \wedge W(a, \hat{v}) = v$$

3. *There exists no value $V^{**} > V^*$ and set $\hat{\hat{\mathcal{V}}}$ that satisfies properties 1 and 2; furthermore, there is no set $\hat{\mathcal{V}}_a \supset \hat{\mathcal{V}}$ that satisfies properties 1 and 2 together with V^*.*

Then:

- *Construct an arbitrary sequence of actions $\{a_t^*\}_{t=0}^\infty$ recursively as follows. In period 0, pick $\hat{v}_0^* \in \hat{\mathcal{V}}$*

[12] Note, however, that we cannot adopt their method to recursively compute the desired sets. *Given* V^*, $\hat{\mathcal{V}}$ can be computed recursively as in Abreu, Pierce, and Stacchetti. However, without further assumptions the set of values of V^* for which $\hat{\mathcal{V}}$ is defined need not be convex, which makes finding its maximum difficult.

and $(a_0^*, \hat{v}_1^*) \in A \times \hat{\mathcal{V}}$ *such that* $\tilde{V}(a_0^*, \hat{v}_1^*) = V^*$ *and* $W(a_0^*, \hat{v}_1^*) = \hat{v}_0^*$. *In each subsequent period, pick* $(a_t^*, \hat{v}_{t+1}^*) \in A \times \hat{\mathcal{V}}$ *such that* $\tilde{V}(a_t^*, \hat{v}_{t+1}^*) = V^*$ *and* $W(a_t^*, \hat{v}_{t+1}^*) = \hat{v}_t^*$. *Constructing such a sequence is possible by the definition of* V^* *and* $\hat{\mathcal{V}}$. *The sequence so constructed is the outcome of a reconsideration-proof equilibrium;*

- *If* $\{a_t^*\}_{t=0}^{\infty}$ *is the equilibrium path of a reconsideration-proof equilibrium,* $\tilde{V}(a_0^*, a_1^*, ...) = V^*$ *and* $\hat{V}(a_t^*, a_{t+1}^*, ...) \in \hat{\mathcal{V}}$ *for any* $t > 0$.

Proof.

First, we prove that the recursively-constructed sequence $\{a_t^*\}_{t=0}^{\infty}$ satisfies

$$\tilde{V}(a_t^*, \hat{V}(a_{t+1}^*, a_{t+2}^*, ...)) = V^* \quad \forall t \geq 0 \tag{B.1}$$

and

$$\widehat{V}(a_t^*, a_{t+1}^*, a_{t+2}^*, ...) \in \hat{\mathcal{V}} \quad \forall t \geq 0. \tag{B.2}$$

Note that, if $\widehat{v}_T^* = \widehat{V}(a_{T+1}^*, a_{T+2}^*, a_{T+3}^*, ...)$ for some period T, iterating backwards we find that $\widehat{v}_t^* = \widehat{V}(a_{t+1}^*, a_{t+2}^*, a_{t+3}^*, ...)$ for all $t < T$[13], so that equations (B.1) and (B.2) hold.

Define

$$\{\underline{a}_t\}_{t=0}^{\infty} \in \arg\min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)$$

and similarly let $\{\bar{a}_t\}_{t=0}^{\infty}$ be a sequence that attains the maximum. Both exist by the compactness of A and the continuity of $\widehat{V}$ (in the product topology).

Next, truncate the sequence $\{a_t^*\}_{t=0}^{\infty}$ at time $S > T$ and replace the continuation with $\{\underline{a}_t\}_{t=0}^{\infty}$ or $\{\bar{a}_t\}_{t=0}^{\infty}$. By assumption 4 and the monotonicity of W, we have

$$\widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \underline{a}_0, \underline{a}_1, ...) \leq \widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, a_{S+1}^*, a_{S+2}^*, ...) \leq \widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \bar{a}_0, \bar{a}_1, ...) \tag{B.3}$$

[13] Should it be $\widehat{v}_T^* = \widehat{V}(a_T^*, a_{T+1}^*, a_{T+2}^*, a_{T+3}^*, ...)$ and $\widehat{v}_t^* = \widehat{V}(a_t^*, a_{t+1}^*, a_{t+2}^*, a_{t+3}^*, ...)$?

and

$$
\begin{aligned}
&\widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \underline{a}_0, \underline{a}_1, ...) = W(a_T^*, W(a_{T+1}^*, ...W(a_S^*, W(\underline{a}_0, W(\underline{a}_1, ...)...))...)) \leq \\
&W(a_T^*, W(a_{T+1}^*, ...W(a_S^*, \hat{v}_S^*)...)) = \hat{v}_T^* \leq \\
&W(a_T^*, W(a_{T+1}^*, ...W(a_S^*, W(\bar{a}_0, W(\bar{a}_1, ...)...))...)) = \widehat{V}(a_T^*, a_{T+1}^*, ..., a_S^*, \bar{a}_0, \bar{a}_1, ...)
\end{aligned}
\tag{B.4}
$$

Taking limits as $T \to \infty$ in equations (B.3) and (B.4) and exploiting the continuity of $\widehat{V}$ according to the product topology, the left-most and right-most expressions in the inequalities converge to the same value, which then implies that indeed $\widehat{v}_T^* = \widehat{V}(a_{T+1}^*, a_{T+2}^*, a_{T+3}^*, ...)$ and (B.1) and (B.2) hold.

To complete the proof of the first point, we need to show that there exists no symmetric subgame-perfect equilibrium whose payoff is strictly greater than V^*. By contradiction, suppose that there is such an equilibrium with value $V^{**} > V^*$. Let σ^{**} be the strategy profile representing one such equilibrium. Define

$$\hat{\mathcal{V}}_b := \{v : v = \widehat{V}(a_{t+1}^{**}|_{h^t}, a_{t+2}^{**}|_{h^t}, a_{t+3}^{**}|_{h^t}, ...),\ h^t \in A^t\},$$

where $\{a_s^{**}|_{h^t}\}_{s=t+1}^{\infty}$ is the equilibrium path implied by the strategy profile σ^{**} following a history h^t. The pair $(V^{**}, \hat{\mathcal{V}}_b)$ satisfies property 1 in the lemma, since otherwise σ_0^{**} would not be optimal at time 0. It also satisfies property 2 since σ^{**} is symmetric and by the definition of $\hat{\mathcal{V}}_b$. But then this implies that property 3 in the lemma does not hold for V^*, establishing a contradiction.

In the previous point we proved that, given V^* and $\hat{\mathcal{V}}$, we can construct a reconsideration-proof equilibrium of value V^*. Since all reconsideration-proof equilibria must have the same value, it must be the case that $\tilde{V}(a_0^*, a_1^*, ...) = V^*$. Furthermore, repeating the steps of the previous point, we can prove that the value V^* and the set

$$\hat{\mathcal{V}}_a := \{v : v = \widehat{V}(a_{t+1}^*|_{h^t}, a_{t+2}^*|_{h^t}, a_{t+3}^*|_{h^t}, ...),\ h^t \in A^t\},$$

satisfy properties 1 and 2. By the definition of $\hat{\mathcal{V}}$, it follows that $\hat{\mathcal{V}}_a \subseteq \hat{\mathcal{V}}$. □

While not essential for the proof of Proposition 4, the following lemma is useful for computations:

Lemma 4. *The set $\hat{\mathcal{V}}$ defined in Lemma 3 is convex.*[14]

[14] Lemma 3 defines a unique set, since the union of all sets satisfying properties 1 and 2 satisfies properties 1 and 2 as well.

Proof. We first define the set $\hat{\mathcal{V}}_c$ by relaxing property 2 in Lemma 3 to be the following:

$$\forall v \in \hat{\mathcal{V}}_c \quad \exists (a, \hat{v}) \in A \times \hat{\mathcal{V}} : \tilde{V}(a, \hat{v}) \geq V^* \wedge W(a, \hat{v}) = v. \tag{B.5}$$

We will later prove that $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$.

Simple case. First, if $\hat{\mathcal{V}}_c$ is a singleton, then it is necessarily convex and $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$: by property 3 of Lemma 3, V^* should be raised until $\tilde{V}(a, \hat{v}) = V^*$ at the single element $\hat{v} \in \hat{\mathcal{V}}_c$, with no effect on property 2 and relaxing the constraint in property 1.

From now on, we study the case in which $\hat{\mathcal{V}}_c$ contains at least two values.

Step 1. To prove that $\hat{\mathcal{V}}_c$ is convex, we prove that its convex hull, $\mathrm{Co}(\hat{\mathcal{V}}_c)$, satisfies properties 1 and 2 as well (and of course $\mathrm{Co}(\hat{\mathcal{V}}_c) \supset \hat{\mathcal{V}}_c$ unless $\hat{\mathcal{V}}_c$ is convex as well). Property 1 is immediate from the monotonicity of $\tilde{V}$. Let $v_1, v_2 \in \hat{\mathcal{V}}_c$, and let $(a_1, \hat{v}_1)$, $(a_2, \hat{v}_2)$ elements of $A \times \hat{\mathcal{V}}_c$ be two pairs of actions and continuation values that satisfy property 2 of Lemma 3. Consider their convex combination $(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2)$, $\alpha \in [0, 1]$. Since $\tilde{V}$ is continuous and quasiconcave and W is continuous, $\tilde{V}(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2) \geq V^*$, and $W(\alpha v_1 + (1-\alpha) v_2, \alpha \hat{v}_1 + (1-\alpha)\hat{v}_2)$ takes all values in $[v_1, v_2]$ as α varies between 0 and 1. Hence, all intermediate values satisfy property 2 as well, which completes the proof that $\mathrm{Co}(\hat{\mathcal{V}}_c)$ satisfies property 2.

Step 2. To prove that $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$, proceed as follows. Define $\underline{v}_c := \min\{\hat{\mathcal{V}}_c\}$ and $\bar{v}_c := \max\{\hat{\mathcal{V}}_c\}$.[15] By definition, we can find $(\underline{a}, \hat{\underline{v}})$ and $(\bar{a}, \bar{\hat{v}})$ such that

$$\tilde{V}(\underline{a}, \hat{\underline{v}}) \geq V^* \wedge W(\underline{a}, \hat{\underline{v}}) = \underline{v}$$

and

$$\tilde{V}(\bar{a}, \bar{\hat{v}}) \geq V^* \wedge W(\bar{a}, \bar{\hat{v}}) = \bar{v}.$$

Since A is convex, we can construct within it a line from $\underline{a}$ to $\bar{a}$ by defining $a(\alpha) := \alpha \underline{a} + (1-\alpha)\bar{a}$, $\alpha \in [0, 1]$. By the quasiconcavity of $\tilde{V}$, we know

$$\tilde{V}(a(\alpha), \alpha \hat{\underline{v}} + (1-\alpha)\bar{\hat{v}}) \geq V^*.$$

[15] It is straightforward to prove that $\hat{\mathcal{V}}_c$ is closed, by the continuity of the functions defining it.

By property 1 of Lemma 3, for each action $a(\alpha)$ and the monotonicity and continuity of $\tilde{V}$ we have

$$\tilde{V}(a(\alpha), \underline{v}) \leq V^*.$$

Since $\hat{\mathcal{V}}_c$ is convex, we can find a (unique) value $\hat{v}(\alpha)$ such that

$$\tilde{V}(a(\alpha), \hat{v}(\alpha)) = V^*.$$

Monotonicity and continuity of $\tilde{V}$ imply that $\hat{v}(\alpha)$ is a continuous function. It then follows that $\hat{V}(a(\alpha), \hat{v}(\alpha))$ is a continuous function of α. As $\alpha \in [0,1]$, this function must take all values between $\underline{v}$ and $\bar{v}$, proving that the property 2 of Lemma 3 is satisfied by $\hat{\mathcal{V}}_c$ and thus $\hat{\mathcal{V}}_c = \hat{\mathcal{V}}$. □

We are now ready to prove Proposition 4.

Proof. The second property of the value V^* and the set $\hat{V}$ in Lemma 3 implies that we can construct a function $g : \hat{\mathcal{V}} \to \mathbb{R} \times \hat{\mathcal{V}}$ with the property that $\tilde{V}(g(v)) = V^*$ and $W(g(v)) = v$.[16] Starting from any value $v_0 \in \hat{\mathcal{V}}$, we can construct recursively a path $(a_t, v_{t+1}) = g(v_t)$. By Lemma 3, this is the equilibrium path of a reconsideration-proof equilibrium. It will thus be an organizational equilibrium provided that

$$V(a_t, v_{t+1}) \geq \max_a \tilde{V}(a, v_0) \quad \forall t.$$

By the definition of $\mathcal{V}$, this property is satisfied by its least element, $\underline{v}$;[17] hence, it will be satisfied provided that the initial value v_0 is sufficiently low. □

We now proceed to prove Proposition 5.

Proof. Define a correspondence $\zeta : \mathbb{R} \times \mathbb{R} \rightrightarrows \mathbb{R}$ as follows:

$$v \in \zeta(v', v^*) \iff \exists a \in A : \begin{cases} \tilde{V}(a, v') = v^* \\ W(a, v') = v. \end{cases} \tag{B.6}$$

[16] This function may not be unique.

[17] By the monotonically of $\widetilde{V}$ in its second argument and the property 1 of $\mathcal{V}$, $\tilde{V}(a, \underline{v}) \leq V^*$ for all $a \in A$.

In words, given (v^*, v'), v belongs to the correspondence if there is an action a which, together with a continuation value v', yields utility v^* when evaluated according to the decision maker's preferences $(\tilde{V})$ and utility v when evaluated with her continuation utility function W.

We prove that there exists a value v^* for which ζ is nonempty and admits a fixed point in continuation utilities $(v = v')$. We do so by proving that a Markov equilibium (a^M, v^M) exists, such that[18]

$$V^* = \tilde{V}(a^M, v^M) = \max_a \tilde{V}(a, v^M) \tag{B.7}$$

and

$$v^M = W(a^M, v^M). \tag{B.8}$$

To prove the existence of a Markov equilibrium, we construct a correspondence $\hat{a}(.)$ from A into itself by setting

$$\hat{a}(a) = \max_{a_0 \in A} \widehat{V}(a_0, a, a, a, ...).$$

By the usual compactness and continuity properties, this correspondence is nonempty, compact-valued, and upper hemicontinuous. Quasiconcavity of $\widehat{V}$ ensures that it is also convex-valued. Hence, the correspondence has a fixed point by Kakutani's theorem; let a^M be one such fixed point. Given Assumption 4, letting $v^M := \widehat{V}(a^M, a^M, a^M, ...)$, equations (B.7) and (B.8) are satisfied.

We thus know $v^M \in \zeta(v^M, \tilde{V}(a^M, v^M))$. Once again, our assumptions about compactness and continuity imply that the correspondence ζ is upper hemicontinuous. Let V^* be the maximal value for which ζ admits a fixed point in continuation utilities. In the proofs below, it is useful to establish that

$$v \in \zeta(v', V^*) \Longrightarrow v \leq v'. \tag{B.9}$$

Suppose (B.9) is not satisfied. Let (a, v') be such that $V(a, v') = V^*$ and $W(a, v') > v'$. Holding the action a fixed, continuity and monotonicity imply that higher values of v' lead to higher values of $V(a, v')$ and $W(a, v')$. As long as $W(a, v') > v'$, we know that $v' < \max_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)$ and can thus be raised further. Eventually, we will attain a value $v^h > v'$ for which $W(a, v^h) = v^h$ (this has to happen, since $W(a, v')$ is bounded by the maximum above). Let $V^h := V(a, v^h) > V^*$. We just established that a fixed point of $\zeta(., V^h)$ exists, which contradicts the assumption that V^* is the highest value for which a fixed point can be found.

[18] Should it be v^* instead of V^* in equation (B.7)?

In our next step, we prove that there are no symmetric equilibria with value $V^{**} > V^*$. By the definition of V^*, given any combination of an action and a continuation utility (a, v'), if $\tilde{V}(a, v') = V^{**}$ then $W(a, v') < v'$. This implies that any equilibrium path with value V^{**} would feature a strictly increasing sequence of continuation values; convergence is ruled out, because continuity and compactness would imply that the limiting point would be a fixed point of ζ, which is inconsistent with $V^{**} > V^*$. Since the set of possible continuation values is bound by

$$\max_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, \ldots),$$

no such equilibrium path can exist.

We now prove that there exist symmetric equilibria with value V^*, which then implies that any such equilibrium is reconsideration proof. Let v^{SS} be the maximal fixed point of $\zeta(., V^*)$. For any continuation value $v > v^{SS}$, a repetition of the arguments described above for V^{**} imply that no equilibrium path would be possible.[19] We prove instead that there exists a convex set $\mathcal{V} = [v_\ell, v^{SS}]$ which, together with V^*, satisfies the properties of Lemma 3, where

$$v_\ell := \min_{v' \leq v^{SS}} \min \zeta(v', V^*). \tag{B.10}$$

To do so, prove first that, for any action $a \in A$, $\tilde{V}(a, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) \leq V^*$. By contradiction, suppose that an action a_L such that $\tilde{V}(a_L, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) > V^*$ existed. We could then repeat the same steps used to prove (B.9) and construct a steady state with value higher than V^*.

Since $\tilde{V}(a, \min_{\{a_t\}_{t=0}^{\infty}} \widehat{V}(a_0, a_1, ...)) \leq V^* \quad \forall a \in A$, we can define

$$v'_{\min} := \min_{(a,v')} v' := \tilde{V}(a, v') = V^*.$$

Since there exists an action a^{SS} such that $V(a^{SS}, v^{SS}) = V^*$, $v'_{\min} \leq v^{SS}$. Also, by equations (B.9) and (B.10), $v_\ell \leq v_{\min}$[20]. Hence, $V(a, v_\ell) \leq V^* \quad \forall a \in A$[21]: Property 1 of Lemma 3 is satisfied by the value V^* and the continuation set $[v_\ell, v^{SS}]$. To prove Property 2, let a_ℓ and v'_ℓ be such that $W(a_\ell, v'_\ell) = v_\ell$, and $\lambda \in [0, 1]$[22]. As we just established, $\tilde{V}(\lambda a_\ell + (1-\lambda)a^{SS}, v_\ell) \leq V^*$. By quasiconcavity, $\tilde{V}(\lambda a_\ell + (1-\lambda)a^{SS}, \lambda v'_\ell + (1-\lambda)v^{SS}) \geq V^*$.

[19] If along the equilibrium path, for some $T \geq 0$, $v_T > v^{SS}$, then $v_t > v^{SS}$ for all $t > T$. Since $\{v_t\}$ is bounded and monotonically increasing, the limiting point will be a fixed point of ζ, which is a contradiction to that v^{SS} is the largest fixed point.

[20] Should here be $v'_{\min}$ instead of $v_{\min}$? Is the prime supposed to make it different from $\underline{v} \equiv \min_{\{a_t\}} \widehat{V}(a_0, a_1, \ldots)$?

[21] Should it be $\widetilde{V}(a, v_\ell)$ and $\widetilde{V}(a^{SS}, v^{SS})$?

[22] Should we choose a_ℓ and v'_ℓ such that $W(a_\ell, v'_\ell) = v_\ell$ and $\widetilde{V}(a_\ell, v'_\ell) = V^*$? Is it possible that $v'_\ell > v^{SS}$?

Strict monotonicity implies that there exists a unique value v_λ such that $\tilde{V}(\lambda a^{SS} + (1-\lambda)a_\ell, v_\lambda) = V^*$, which must vary continuously with λ by the continuity of $\tilde{V}$. It follows that $W(\lambda a^{SS} + (1-\lambda)a_\ell, v_\lambda)$ is a continuous function of λ and it takes all values between v_ℓ and v^{SS}, proving that Property 2 of Lemma 3 holds. Finally, from equations (B.9) and (B.10), we know that any value $v \notin [v_\ell, v^{SS}]$ could only be attained by some action a with a continuation value $v' > v^{SS}$, which would lead to nonexistence in subsequent periods. Hence, $[v_\ell, v^{SS}]$ is the largest set that satisfies Properties 1 and 2 of Lemma 3 together with the value V^*, completing the proof that a reconsideration-proof equilibrium has value V^*, and thus that in turn the organizational equilibrium with the state variable is also associated with an action value V^*.

Finally, suppose that V is strictly quasiconcave. Let a^{SS} be the unique action that attains $\max_a V(a, a, a, ...)$. If this steady state is not a Markov equilibrium, then $a^{SS} < \max_a \tilde{V}(a, v^{SS})$. In this case, a sequence that starts at a^{SS} and stays constant violates the no-delay condition. □

C Approximated Equilibrium

Consider the class of economies specified in Section 4.1.

$$\Psi_t = u(k_t, a_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u\left(k_{t+\tau}, a_{t+\tau}\right). \tag{C.1}$$

such that

$$u(k, a) = C_{10} + C_{11} h(k) + C_{12} m(a) \tag{C.2}$$

$$h(k') = C_{20} + C_{21} h(k) + C_{22} g(a) \tag{C.3}$$

In the approximated equilibrium, C is a constant matrix chosen to match the level and first order derivative.

$$C = \begin{bmatrix} u(\bar{k}, \bar{a}) - \frac{u_k(\bar{k},\bar{a})}{h_k(\bar{k})} h(\bar{k}) - \frac{u_a(\bar{k},\bar{a})}{m_a(\bar{a})} m(\bar{a}) & \frac{u_k(\bar{k},\bar{a})}{h_k(\bar{k})} & \frac{u_a(\bar{k},\bar{a})}{m_a(\bar{a})} \\ h(\bar{k}) - F_k(\bar{k}, \bar{a}) h(\bar{k}) - \frac{h_k(\bar{k}) F_a(\bar{k},\bar{a})}{g_a(\bar{a})} g(\bar{a}) & F_k(\bar{k}, \bar{a}) & \frac{h_k(\bar{k}) F_a(\bar{k},\bar{a})}{g_a(\bar{a})}. \end{bmatrix} \tag{C.4}$$

It is easy to verify the weakly separable utility. Supposing the sequence of rescaled actions is $\{a_\tau\}_{\tau=0}^{\infty}$ and the initial state is k_0, then the sequence of state variables is

$$h(k_t) = C_{20}\frac{1 - C_{21}^t}{1 - C_{21}} + C_{21}^t h(k_0) + C_{22}\sum_{\tau=0}^{t-1} C_{21}^{t-1-\tau} g(a_\tau). \tag{C.5}$$

The lifetime utility is given by

$$U(k_0, \{a_\tau\}_{\tau=0}^{\infty}) = \frac{1 - \beta + \delta\beta}{1 - \beta} C_{10} + \frac{C_{11}(1 - \beta C_{21} + \delta\beta C_{21})}{1 - \beta C_{21}} h(k_0) + V(\{a_\tau\}_{\tau=0}^{\infty}), \tag{C.6}$$

where

$$V(\{a_\tau\}_{\tau=0}^{\infty}) = C_{12}m(a_0) + \frac{C_{11}C_{22}\delta\beta}{1 - \beta C_{21}} g(a_0) + \delta\sum_{j=1}^{\infty}\beta^j\left(C_{12}m(a_j) + \frac{\beta C_{11}C_{22}}{1 - \beta C_{21}} g(a_j)\right). \tag{C.7}$$

Let $V_t \equiv V(\{a_\tau\}_{\tau=t}^{\infty})$, then

$$V_t = C_{12}m(a_t) + \frac{\delta\beta C_{11}C_{22}}{1 - \beta C_{21}} g(a_t) + \beta V_{t+1} - \beta(1 - \delta)C_{12}m(a_{t+1}). \tag{C.8}$$

In a stationary allocation,

$$(1 - \beta)V = (1 - \beta(1 - \delta))C_{12}m(a) + \frac{\delta\beta C_{11}C_{22}}{1 - \beta C_{21}} g(a). \tag{C.9}$$

In an organizational equilibrium, the interior maximum, a^*, has to equal to the stationary rescaled action $\overline{a}$. Therefore,

$$(1 - \beta(1 - \delta))C_{12}m_a(\overline{a}) + \frac{\delta\beta C_{11}C_{22}}{1 - \beta C_{21}} g_a(\overline{a}) = 0. \tag{C.10}$$

It follows that the stationary allocation has to be satisfy

$$(1 - \beta(1 - \delta))u_a(\overline{k}, \overline{a})(1 - \beta F_k(\overline{k}, \overline{a})) + \delta\beta u_k(\overline{k}, \overline{a})F_a(\overline{k}, \overline{a}) = 0. \tag{C.11}$$

Note that this condition characterizes the stationary allocation and is independent of the choice of $h(k), m(a), g(a)$.

Along the transition, the proposal function $a_{t+1} = q(a_t)$ is given by

$$a_{t+1} = m^{-1}\left\{\frac{C_{12}m(a_t) + \frac{\delta\beta C_{11}C_{22}}{1-\beta C_{21}}g(a_t) - (1-\beta)V^*}{\beta(1-\delta)C_{12}}\right\} \tag{C.12}$$

In terms of the initial starting point, recall that

$$V(\{a_\tau\}_{\tau=0}^{\infty}) = C_{12}m(a_0) + \frac{C_{11}C_{22}\delta\beta}{1-\beta C_{21}}g(a_0) + \delta\sum_{j=1}^{\infty}\beta^j\left(C_{12}m(a_j) + \frac{\beta C_{11}C_{22}}{1-\beta C_{21}}g(a_j)\right) \tag{C.13}$$

If the initial agent take future actions as given, then she will simply maximize $C_{12}m(a_0) + \frac{C_{11}C_{22}\delta\beta}{1-\beta C_{21}}g(a_0)$, which leads to the Markov action a^M that solves

$$C_{12}m_a(a^M) + \frac{C_{11}C_{22}\delta\beta}{1-\beta C_{21}}g_a(a^M) = 0 \tag{C.14}$$

To make sure the initial agent is willing to make a proposal, it has to be that

$$a_0 = q(a^M). \tag{C.15}$$

D Taxation Section

D.1 A Description of the Game for Policy Applications

In Section 3, there is one player for each period. Here, the policymaker is still represented by one player for each period, but we also include a continuum of identical households that face a dynamic problem.[23]

The game unfolds as follows. In each period, the government in power takes an action $a \in A$ first. Then, the households move simultaneously. Each household takes an action $s \in S$. The aggregate state for next period evolves according to $k' = F(k, a, s)$. A full description would require us to specify what happens when households take different actions, so that, while they are identical ex ante, they may end up being different ex post. However, in most of the applications that are of interest, the household optimization problem has a unique solution. Hence, there can be no equilibrium in which identical households take different actions.

[23] The notion of an equilibrium can be readily extended to environments with finite types of households, or to economies with overlapping generations. Extending organizational equilibrium to environments can be done by interacting the analysis here with distributional notions of equilibrium as in Jovanovic and Rosenthal (1988).

Moreover, a deviation from a single household has no effect on aggregates. We exploit these properties and specify the evolution of the economy and preferences only after histories in which (almost) all households have taken the same action. Starting from an arbitrary period t and state k_t, household preferences are given by a function

$$Z(k_t, \{a_v, s_v, s_v^-\}_{v=t}^{\infty}), \tag{D.1}$$

where s_v represents the action taken by the individual household, and s_v^- is the action taken by (almost) all other households. We assume that S is a convex compact subset of a locally convex topological linear space and that Z is jointly continuous in all of its arguments (in the product topology), strictly quasiconcave in the own action sequence $\{s_v\}_{v=t}^{\infty}$, and weakly separable between the state and the remaining arguments. We also assume that household preferences are time consistent. More precisely, we assume that, given an initial level of the state k_t and a sequence of other households' actions $\{a_v, s_v\}_{v=t}^{\infty}$,

$$\begin{aligned} &Z(k_t, \{a_v, s_v, s_v\}_{v=t}^{\infty}) = \max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty}) \Longrightarrow Z(F(k_t, a_t, s_t), \{a_v, s_v, s_v\}_{v=t+1}^{\infty}) = \\ &\max_{\{\tilde{s}_v\}_{v=t+1}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t+1}^{\infty}). \end{aligned} \tag{D.2}$$

Equation (D.2) states that, if it is optimal from period t to follow the same sequence of actions that all other households are taking, then it is also optimal to follow that sequence also in subsequent periods, as long as other households also continue to do the same. Notice that we exploit the fact that each household has no effect on the aggregates to leave the continuation preferences over several histories unspecified; this is convenient, because it prevents us from having to explicitly introduce individual state variables. To be concrete, consider the taxation game to which we apply this general definition; in that game, s_t is the individual saving rate. Equation (D.2) is written from the perspective of a household that starts with the same level of k_t as the aggregate, which allows us not to draw a distinction between the two. If that household finds it optimal to follow the same saving rate as all other households, then it will optimally choose to have the same level of k_{t+1}, and equation (D.2) ensures that the continuation plan will remain optimal from period $t+1$ onwards. If instead the household chooses a different saving rate from others, then it would potentially enter period $t+1$ with a different level of the state from the aggregate; however, whenever this choice does not maximize (D.1), we know this would not be an optimal individual choice without need to specify the entire continuation path; moreover, the individual deviation does not affect aggregate incentives, hence we do not need to keep track of it for the purpose of computing other households' best response either.

We define a competitive equilibrium from period t and a state k_t as a sequence $\{a_v, s_v\}_{v=t}^{\infty}$ such that

$$Z(k_t, \{a_v, s_v, s_v\}_{v=t}^{\infty}) = \max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty}).$$

Proposition 7. *Given any sequence of policy actions $\{a_v\}_{v=t}^{\infty}$, a competitive equilibrium exists.*

Proof. Fix k_t and $\{a_v\}_{v=t}^{\infty}$. Given our assumptions on S and Z, the best-response function

$$br(\{s_v\}_{v=0}^{\infty}) := \arg\max_{\{\tilde{s}_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v\}_{v=t}^{\infty})$$

is well defined and continuous. By Brouwer's theorem, it admits a fixed point, which is a competitive equilibrium. □

Equation (D.2) ensures that the continuation of a competitive equilibrium is a competitive equilibrium itself. In what follows, we proceed by assuming that the competitive equilibrium is unique given the policy action, which can be verified in each specific application. Non-uniqueness can be accommodated by assuming a selection rule on how households coordinate when multiple equilibria are possible, as long as this rule has the property that the continuation of a selected competitive equilibrium is selected itself as a continuation competitive equilibrium.

At time t, government preferences are given by a function $\Psi(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots)$.

A symmetric history of play is a record of all actions taken in the past; we distinguish between histories at which the government is called to play, which are given by $h^0 := \emptyset$ and

$$h^t := (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}), \quad t > 0,$$

and histories at which households are called to play, that take the form of $h^{p,0} := a_0$ and

$$h^{p,t} := (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}, a_t), \quad t > 0.$$

Let H be the set of histories at which the government is called to play, and H^p the set of histories at which households are called to play. For the reasons discussed above, we only keep track of histories in which almost all households have taken the same action.

A strategy for the households is a mapping $\sigma^p : H^p \to S$; likewise, a government strategy is a mapping $\sigma : H \to A$. A *symmetric strategy profile* is a pair (σ^p, σ), representing how all households and the government will act following any symmetric history; it recursively induces a path of play $\{a_t, s_t\}_{t=0}^{\infty}$.

A symmetric strategy profile (σ^p, σ) is a sequential equilibrium if the following is true:

- Given that the government will follow σ and other households will follow σ^p, the actions dictated by σ^p are optimal for each household. After any history $h^{p,t}$, each household takes as given the government policy action a_t and the initial state k_t, which is recursively determined by the history of past play. Moreover, the strategy σ^p followed by other households and the government strategy σ determine the *future* path of aggregate play, $\{s_v, a_{v+1}\}_{v=t}^{\infty}$. Household optimality requires that the sequence of actions prescribed by σ^p is optimal along this path: equivalently stated, it requires the actions prescribed by σ^p to be a competitive equilibrium from period t on, following any arbitrary (symmetric) history.
- Given that households will follow the strategy σ^p and that future governments will follow the strategy σ, and given any past history h^t, the current government choice $\sigma(h^t)$ is optimal.

Proposition 8. *There exists a sequential equilibrium of the game, in which the payoff from the weakly separable part is independent of the past.*

[to be completed]

D.2 Proof of Lemma 1

First consider the following social planer's problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to the resource constraint

$$c_t + k_{t+1} = k_t^{\alpha_t}.$$

Note that α_t in the production function can be time-varying in a deterministic fashion. The Euler condition is

$$\frac{1}{c_t} = \alpha_{t+1}\beta k_{t+1}^{\alpha_{t+1}-1}\frac{1}{c_{t+1}}.$$

Let μ_t denote the saving rate, i.e., $k_{t+1} = \mu_t k_t^{\alpha_t}$, then the Euler condition above can be rewritten as

$$\frac{1}{(1-\mu_t)z_t k_t^{\alpha_t}} = \alpha_{t+1}\beta k_{t+1}^{\alpha_{t+1}-1}\frac{1}{(1-\mu_{t+1})k_{t+1}^{\alpha_{t+1}}},$$

which can be further simplified to

$$\frac{\mu_t}{(1-\mu_t)} = \alpha_{t+1}\beta\frac{1}{(1-\mu_{t+1})}$$

The associated transversality condition is

$$\lim_{t\to\infty}\beta^t\frac{k_{t+1}}{c_t} = \lim_{t\to\infty}\beta^t\frac{\mu_t}{(1-\mu_t)} = 0.$$

By the standard concavity arguments, the planning problem has a unique solution and the Euler condition and the transaversality condition are necessary and sufficient for optimality. Hence, there must be a unique sequence of saving rates that satisfies them.

Now consider the tax-distorted competitive equilibrium in Section. In the tax-distorted competitive equilibrium, define φ_t as the after taxation saving rate, i.e., $k_{t+1} = \varphi_t(1-\alpha\tau_t)k_t^{\alpha}$, the Euler condition for households is

$$\frac{1}{c_t} = \alpha\beta k_{t+1}^{\alpha-1}\frac{(1-\tau_{t+1})}{c_{t+1}}$$

or

$$\frac{1}{(1-\varphi_t)(1-\alpha\tau_t)k_t^{\alpha}} = \alpha\beta k_{t+1}^{\alpha-1}\frac{(1-\tau_{t+1})}{(1-\varphi_{t+1})(1-\alpha\tau_{t+1})\varphi_t(1-\alpha\tau_t)k_t^{\alpha}}$$

which can be simplified to

$$\frac{\varphi_t}{(1-\varphi_t)} = \alpha\frac{1-\tau_{t+1}}{1-\alpha\tau_{t+1}}\beta\frac{1}{(1-\varphi_{t+1})}$$

The transversality condition is

$$\lim_{t\to\infty}\beta^t\frac{k_{t+1}}{c_t} = \lim_{t\to\infty}\beta^t\frac{\varphi_t}{(1-\varphi_t)} = 0$$

The Euler and the transversality condition must hold in the competitive equilibrium of the original economy. By defining $\alpha_t = \frac{1-\tau_{t+1}}{1-\alpha\tau_{t+1}}$, there exists a unique sequence of saving rates that satisfies them in the social planner's problem. As a result, there exists a unique competitive equilibrium.

D.3 Proof of Lemma 2

In an organizational equilibrium, the action payoff to government in different periods should be the same. Therefore, for some constant $\overline{P}$,

$$
\begin{aligned}
\overline{P} = & \sum_{j=0}^{\infty} \beta^j \left\{ \log(1 - \alpha\tau_{t+j} - s_{t+j}) + \gamma \log \tau_{t+j} + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_{t+j} \right\} \\
= & \log(1 - \alpha\tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_t \\
& + \beta \sum_{j=0}^{\infty} \beta^j \left\{ \log(1 - \alpha\tau_{t+j+1} - s_{t+j+1}) + \gamma \log \tau_{t+j+1} + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_{t+j+1} \right\} \\
= & \log(1 - \alpha\tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_t + \beta\overline{P}
\end{aligned}
$$

Define $\overline{V} \equiv (1-\beta)\overline{P}$, it follows that for all t,

$$
\log(1 - \alpha\tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s_t = \overline{V}
$$

This leads to condition (5.6) and (5.7). In addition, the Euler condition for consumers needs to be satisfied, which leads to condition (5.8).

D.4 Proof of Proposition 6

Equation (5.10) simply rewrites condition (5.8) using the saving rate defined in equation (5.9). By Lemma (2), the sequence of tax rates derived from $q(\tau)$ together with the sequence of saving rates derived from $h(\tau; V^*)$ satisfy the Euler equation and that the action payoff to government in different periods is equalized. If $q(\tau)$ does not have a fixed point, then the tax rate will diverge to the upper or lower bound, which cannot be an equilibrium. If $q(\tau)$ has a fixed point, then V^* is equal to the highest payoff in the steady state. Therefore, V^* solves

$$
V^* = \max_{\tau, s} \log(1 - \alpha\tau - s) + \gamma \log \tau + \frac{\alpha\beta(1+\gamma)}{1-\alpha\beta} \log s
$$

subject to

$$
s = (1-\tau)\alpha\beta
$$

Any action payoff higher than V^* cannot yield a fixed point for $q(\tau)$. The initial τ_0 is determined by the no-waiting condition in a straightforward way.

D.5 Ramsey Problem and Markov Equilibrium in the Taxation Problem

In this appendix, we describe the case without capital depreciation deduction.[24] Let s_t denote the saving rate, i.e., $k_{t+1} = s_t f(k_t, \ell_t)$. The allocation in the competitive equilibrium is

$$\begin{aligned} k_{t+1} &= \overline{k} k_t^{1-\delta}(s_t y_t)^{\delta}, \\ g_t &= (\alpha(\tau_t^k + \tau_t) + (1-\alpha)(\tau_t^{\ell} + \tau_t)) y_t, \\ c_t &= (1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^{\ell} + \tau_t)) y_t. \end{aligned}$$

where $y_t = f(k_t, \ell_t)$ is the total output. The household's inter and intra Euler conditions satisfy

$$\begin{aligned} \frac{u_c(t)}{\delta \frac{k_{t+1}}{i_t}} =& \beta u_c(t+1) \left\{ (1 - \tau_{t+1} - \tau_{t+1}^k) f_k(k_{t+1}, \ell_{t+1}) + \frac{1-\delta}{\delta} \frac{i_{t+1}}{k_{t+1}} \right\}, \\ u_{\ell}(t) =& - u_c(t)(1 - \tau_t^{\ell} - \tau_t) f_{\ell}(k_t, \ell_t), \end{aligned}$$

which can be written as

$$\begin{aligned} s_t &= \mu_2 \beta \frac{1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^{\ell} + \tau_t)}{1 - s_{t+1} - \alpha(\tau_{t+1}^k + \tau_{t+1}) - (1-\alpha)(\tau_{t+1}^{\ell} + \tau_t)} \left\{ \alpha(1 - \tau_{t+1}^k - \tau_{t+1}) + s_{t+1} \frac{\mu_1}{\mu_2} \right\}, \\ \gamma_{\ell} \frac{\ell_t}{1 - \ell_t} &= \gamma_c \frac{(1-\alpha)(1 - \tau_t^{\ell} - \tau_t)}{1 - s_t - \alpha(\tau_t^k + \tau_t) - (1-\alpha)(\tau_t^{\ell} + \tau_t)} \end{aligned}$$

[24] When there is capital depreciation deduction, the households' budget constraint is

$$c_t + i_t = w_t \ell_t + r_t k_t - (\tau_t^{\ell} + \tau_t) w_t \ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k + \tau_t)}{r_t} \right) r_t k_t$$

However, this specification will break the weakly separable property. Instead, we assume that the budget constraint is

$$c_t + i_t = w_t \ell_t + r_t k_t - (\tau_t^{\ell} + \tau_t) w_t \ell_t - \left(\tau_t^k + \tau_t - \frac{\delta(\tau_t^k + \tau_t)}{\overline{r}} \right) r_t k_t$$

where $\overline{r}$ is the steady state interest rate. This specification will reserve the weakly separable property.

Given an initial capital level $k_0 = k$, a sequence of saving rates, and a sequence of labor supply choices, the implied sequence of capital is

$$k_t = k_0^{(1-\delta+\alpha\delta)^t} \Pi_{j=0}^{t-1} s_j^{\delta(1-\delta+\alpha\delta)^{t-1-j}} \ell_j^{(1-\alpha)\delta(1-\delta+\alpha\delta)^{t-1-j}} \overline{k}^{1+(1-\delta+\alpha\delta)+\ldots+(1-\delta+\alpha\delta)^{t-1}}$$

Given a sequence of tax rates, the action payoff for the government is

$$V(\mathbf{s}, \boldsymbol{\tau}, \boldsymbol{\tau}^k, \boldsymbol{\tau}^\ell) = (\gamma_c + \gamma_g) \left\{ \frac{\alpha\beta\mu_2}{1-(\mu_1+\alpha\mu_2)\beta} \sum_{j=0}^{\infty} \beta^j \log s_j + \frac{(1-\alpha)(1-\beta\mu_1)}{1-(\mu_1+\alpha\mu_2)\beta} \sum_{j=0}^{\infty} \beta^j \log \ell_j \right\}$$
$$+ \sum_{j=0}^{\infty} \beta^j \gamma_c \log\left(1 - s_j - \alpha(\tau_j^k + \tau_j) - (1-\alpha)(\tau_j^\ell + \tau_j)\right) + \sum_{j=0}^{\infty} \beta^j \gamma_\ell \log(1-\ell_j) + \sum_{j=0}^{\infty} \beta^j \gamma_g \log\left(\alpha(\tau_j^k + \tau_j) + (1-\alpha)(\tau_j^\ell + \tau_j)\right)$$

Ramsey Outcome Let $g_t = \mu_t f(k_t, \ell_t)$. The government budget constraint requires that

$$\mu_t = \alpha(\tau_t^k + \tau_t) + (1-\alpha)(\tau_t^\ell + \tau_t)$$

Depending on the tax instrument used for financing public spending, it is easy to define the required tax rate as a function of μ_t. Denote $\mathcal{T}^k(\mu), \mathcal{T}^\ell(\mu)$, and $\mathcal{T}(\mu)$ as the capital income, labor income, and total income tax rate to achieve the government spending to output ratio μ.

By the primal approach of the Ramsey problem, the government effectively chooses the sequence of saving rates, labor supply, and government spending to output ratios to maximize the welfare of the initial government

$$\max_{\{s_t\},\{\ell_t\},\{\mu_t\}} \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{\gamma_c + \gamma_g}{1-(1-\delta+\alpha\delta)\beta} \left(\alpha\beta\delta \log s_t + (1-\alpha)(1-\beta(1-\delta)) \log \ell_t\right) + \gamma_c \log(1 - s_t - \mu_t)$$
$$+ \gamma_\ell \log(1-\ell_t) + \gamma_g \log(\mu_t) \Bigg)$$

subject to the corresponding implementability constraint

$$\frac{1}{\beta} \frac{s_t}{1-s_t-\mu_t} = \frac{\delta\alpha\left(1 - (\mathcal{T}^k(\mu_{t+1}) + \mathcal{T}(\mu_{t+1}))\chi\right) + (1-\delta)s_{t+1}}{1-s_{t+1}-\mu_{t+1}},$$
$$\gamma_\ell \frac{\ell_t}{1-\ell_t} = \gamma_c \frac{(1-\alpha)(1-\mathcal{T}^\ell(\mu_t) - \mathcal{T}(\mu_t))}{1-s_t-\mu_t}$$

Markov Equilibrium In the Markov equilibrium, the current government take future government's policy as given. In our setting, this will be that taking future government policy as a constant independent of current policy. Assume future tax rates are $\{\tau_f^k, \tau_f^\ell, \tau_f\}$ and the current policy choice is $\{\tau_0^k, \tau_0^\ell, \tau_0\}$. The current government action payoff is

$$\begin{aligned}
&M(\tau_0^k, \tau_0^\ell, \tau_0; \tau_f^k, \tau_f^\ell, \tau_f) \\
=&(\gamma_c + \gamma_g)\left\{\frac{\alpha\beta\mu_2}{1-(\mu_1+\alpha\mu_2)\beta}\left(\log s_0 + \frac{\beta}{1-\beta}\log s_f\right) + \frac{(1-\alpha)(1-\beta\mu_1)}{1-(\mu_1+\alpha\mu_2)\beta}\left(\log \ell_0 + \frac{\beta}{1-\beta}\log \ell_f\right)\right\} \\
&+\gamma_c\left\{\log\left(1 - s_0 - \alpha(\tau_0^k+\tau_0) - (1-\alpha)(\tau_0^\ell+\tau_0)\right) + \frac{\beta}{1-\beta}\log\left(1 - s_f - \alpha(\tau_f^k+\tau_f) - (1-\alpha)(\tau_f^\ell+\tau_f)\right)\right\} \\
&+\gamma_\ell\left\{\log(1-\ell_0) + \frac{\beta}{1-\beta}\log(1-\ell_f)\right\} \\
&+\gamma_g\left\{\log\left(\alpha(\tau_0^k+\tau_0) + (1-\alpha)(\tau_0^\ell+\tau_0)\right) + \frac{\beta}{1-\beta}\log\left(\alpha(\tau_f^k+\tau_f) + (1-\alpha)(\tau_f^\ell+\tau_f)\right)\right\}
\end{aligned}$$

The current government's problem is

$$\max_{\tau_0^k, \tau_0^\ell, \tau_0} M(\tau_0^k, \tau_0^\ell, \tau_0; \tau_f^k, \tau_f^\ell, \tau_f)$$

subject to the implementability constraints

$$\begin{aligned}
s_0 &= \delta\beta\frac{1 - s_0 - \alpha(\tau_0^k+\tau_0) - (1-\alpha)(\tau_0^\ell+\tau_0)}{1 - s_f - \alpha(\tau_f^k+\tau_f) - (1-\alpha)(\tau_f^\ell+\tau_f)}\left\{\alpha(1-\tau_f^k-\tau_f) + s_f\frac{1-\delta}{\delta}\right\}, \\
\gamma_\ell\frac{\ell_0}{1-\ell_0} &= \gamma_c\frac{(1-\alpha)(1-\tau_0^\ell-\tau_0)}{1 - s_0 - \alpha(\tau_0^k+\tau_0) - (1-\alpha)(\tau_0^\ell+\tau_0)} \\
s_f &= \frac{\delta\alpha\beta(1-\tau_f^k-\tau_f)}{1-\beta(1-\delta)} \\
\gamma_\ell\frac{\ell_f}{1-\ell_f} &= \gamma_c\frac{(1-\alpha)(1-\tau_f^\ell-\tau_f)}{1 - s_f - \alpha(\tau_f^k+\tau_f) - (1-\alpha)(\tau_f^\ell+\tau_f)}
\end{aligned}$$

The Markov equilibrium is then the fixed point where future taxes and the current taxes are the same.